محاضـــرات الدكتـــور ابراهيم محفوظ ابراهيم

# MICRO COMPUTER AIDED ANALYSIS, DESIGN AND INDUSTRILIZATION OF STRUCTURES

BY

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In this work, a series of articles is presented with the objective of introducing and demonstrating the great potential of the use of computers in the analysis, and design of structures, as well as, in achieving automatic optimum minimum weight, or minimum cost design for the industrilized structural components fabricated in large quantities.

The first group of articles is devoted to present the use of micro computers in enhancing and simplifying structural engineering work through the development of design aid tables for certain types of structural elements commonly used by engineers. Sample programs are presented for the design and the development of design aid tables for reinforced concrete structural elements. The first group of articles is also concerned with the introduction of the computer aided analysis of structures. In order that the important idea of the utilization matrix formulation in structural analysis may be set forth with a minimum of confusion, the development of the matrix formulation and the computer program for the analysis of beams by the equation of three movements are presented.

The second group of articles is devoted to introducing the computer oriented structural analysis techniques, namely, the finite element displacement method of structural analysis. The first topic covers a brief history of the development of the finite element method, fcllowed by a second article for the presentation and derivation of the formulation of the finite element displacement analysis of structures such as, plane and space trusses and frames. In addition, computer programes are presented and applied to the analysis of these types of structures.

To end the second group articles, the criteria for the evaluation of commertial software are presented and applied to evaluate a sample of commercially available programs in the various categories of structural analysis and designs.

Finally, the third group of articles is devoted to industrilization of structures. In order to illustrate the structural synthesis concept, or in other words the automated optimum design of a structural system, a case study of the automated minimum cost design of prefabricated prestressed concrete beams is presented.

# الحاسبات الالكترونية الصغيرة واستخدامها في المجالات الهندسية

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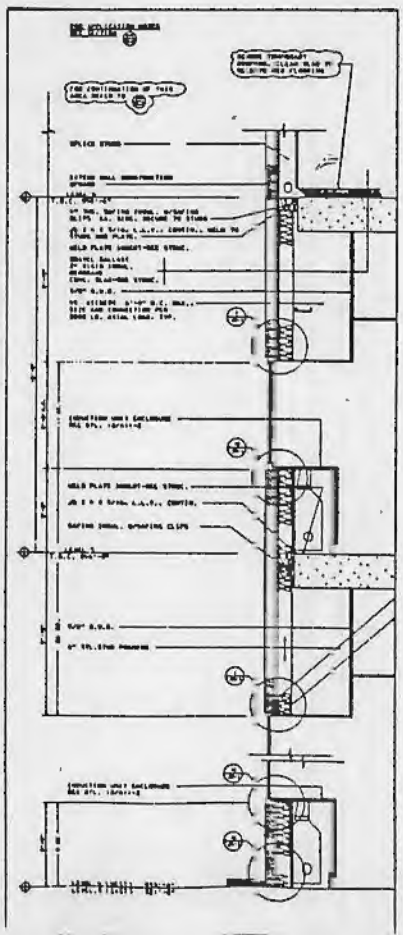
بدأ استخدام الحاسبات الالكترونية الصغيرة في الظهور كعنصر أساسي في جراء الأعمال الإدارية والمالية والهندسية بوجه عام ، وأعمال الهندسة الانشائية بفروعها التصميمية والتنفيذية والتصنيعية بوجه خاص ، وذلك كنتيجة للانخفاض الكبير والمستمر في أسعار شرائها حيث أصبحت الحاسبات الالكترونية في حدود امكانيات كافة الهيئات وكثير من الأفراد . وبناء عليه وجب على المهندسين الممارسين وكذلك على الهيئات العلمية والجامعات أخذ هذا التطور الكبير في الاعتبار مأخذا جديا وسريعا حيث أنه يمكن القول ان « ثورة » استخدامات الحاسبات الالكترونية في المجالات الهندسية قد بدأت علميا بالفعل ، وأنه من المتوقع أن تشمل هذه « الثورة » منطقتنا في القريب العاجل بحيث يمكن القول أنه من غير المستبعد أن الأشخاص والهيئات التي سوف تتجاهل هذه « الثورة » سوف تصل إلى مرحلة من الجمود وعدم الفاعلية في مجالات الممارسة الهندسية .

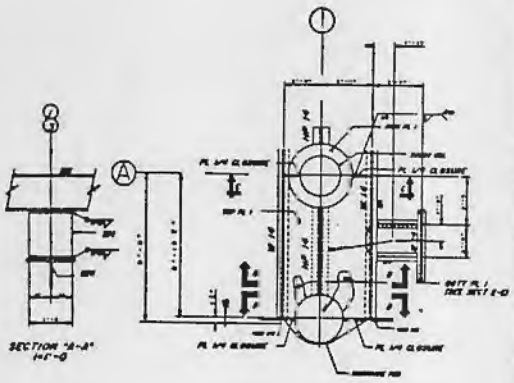
ان استخدام الحاسبات الالكترونية في مجالات الهندسة الانشائية مثلا سوف يتيح للمهندس المصمم التركيز على اختيار أحسن وأنسب نوعية للمنشأ ، وعلى عمل الدراسات لوصول إلى النظم الانشائية الأكثر اقتصاداً ، وكذلك على اعطاء الحلول المرادفة وذلك بدقة وسهولة وسرعة تاركا اجراء العمليات الحسابية المعقدة المستهلكة للوقت وللطاقة البشرية للحاسب الالكتروني مستخدما البرامج الهندسية المختبرة والموثوق منها والتي تلائم طبيعة ونوعية المنشأ المطلوب تصميمه ، هذ بالإضافة إلى السهولة في اجراء التعديلات في التصميم الانشائي التي تتلاءم مع المتطلبات الهندسية الأخرى مثل المتطلبات المعمارية والميكانيكية ... الخ سواء في المراحل الأولية في التصميم أو لأي تعديلات طارئة .

ان تعميم استخدام الحاسبات الالكترونية سوف يتيح للمهندس عمل تصميمات ذات دقة عالية وذلك باستخدام ادق النظريات في كتابة البرامج للحاسب الالكتروني بدون الحاجة إلى عمل الافتراضات التقريبية حتى تتلاءم النظريات مع الحل اليدوي ، هذ بالإضافة إلى الغاء أو التقليل من احتمالات الأخطاء البشرية الممكن حدوثها في اثناء اجراء العمليات الحسابية يدويا هذا مع سهولة أخذ كافة التحميلات والاحتمالات التصميمية في الاعتبار بالاضافة إلى سهولة مراجعة التصميمات ، وكنتيجة لذلك يستطيع المهندس اتمام عمله وهو على ثقة كبيرة من دقة تصميماته .

ان استخدام الحاسبات الالكترونية سوف يفتح المجالات لحل وتصميم نوعيات من المنشآت كان من الصعب أو من المستحيل يدويا حلها لعدم امكانية الوصول إلى انسب الحلول ذات الدقة العالية المطلوبة والتكلفة المقبولة وبطريقة عملية يمكن اتمامها في فترة زمنية معقولة . هذا بالإضافة إلى فتح المجالات لتحسين نظم تنفيذ المشاريع وخصوصا للمشاريع الكبيرة المعقدة والمتشابكة . ان استخدام الحاسبات الالكترونية في مجالات تصنيع المباني سوف يتيح للمصمم المنتج امكانية الوصول « اتوماتيكيا » إلى تصميمات للوحدة الواحدة بحيث يكون وزنها أو تكلفة انتاجها أقل ما يمكن (MINIMUM COST AND MIIMUM WEIGHT DESIGNS) مع استيفاء كافة المتطلبات الانشائية وذلك باستخدام نظرية الحد الأمثل OPTIMIZATION وهذا سوف يكون له كبير الأثر في الاستخدام الأمثل للمواد المختلفة المستخدمة . وفي



تفاصيل معمارية مرسومة بواسطة الحاسب الالكتروني .

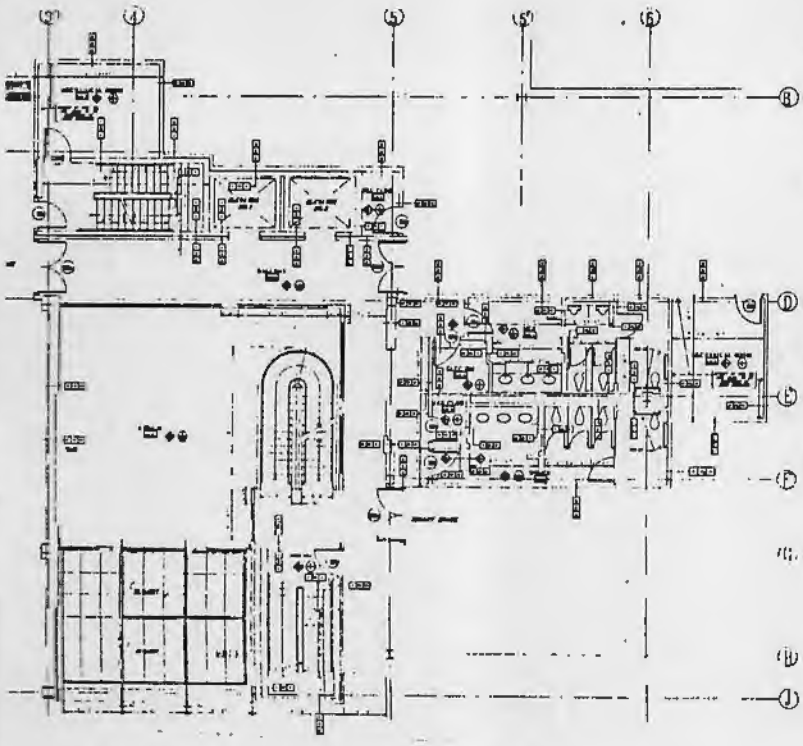


تفاصيل في أجزاء من منشآت معدنية مصممه ومرسومة بواسطة الحاسب الالكتروني .

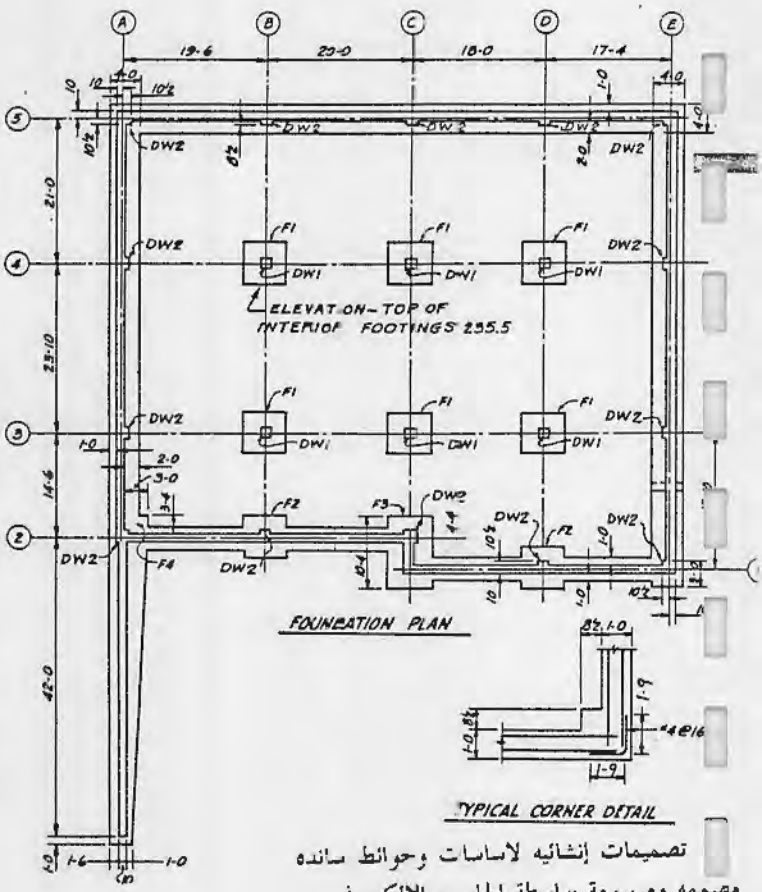
## مقدمـــة :

عون الله تعالى وايمانا من مجلة « عالم البناء » بدورها الرائد في نشر الثقافة المعمارية والهندسية ، تقوم المجلة ابتداء من هذا العدد بنشر سلسلة من المقالات والموضوعات الهندسية التي يعدها مركز الحاسب الالكتروني بمركز الدراسات التخطيطية والمعمارية تحت اشراف الدكتور / ابراهيم محفوظ محمد ابراهيم . وستركز هذه المقالات على تغطية جوانب التعريف بالحاسبات الالكترونية وعناصرها وأسلوب تشغيلها ، كما ستركز أيضا على أعطاء نماذج لأساليب استخدام الحاسب الالكتروني في التصميم الانشائي ، كما سيتم اعطاء جداول تصميمات للمنشآت المختلفة اعدت بواسطة الحاسب الالكتروني ، وتساعد هذه الجداول على تبسيط أعمال التصميم الانشائي وتهم كافة المهندسين والباحثين والدارسين في مجالات العمارة والهندسة الانشائية .

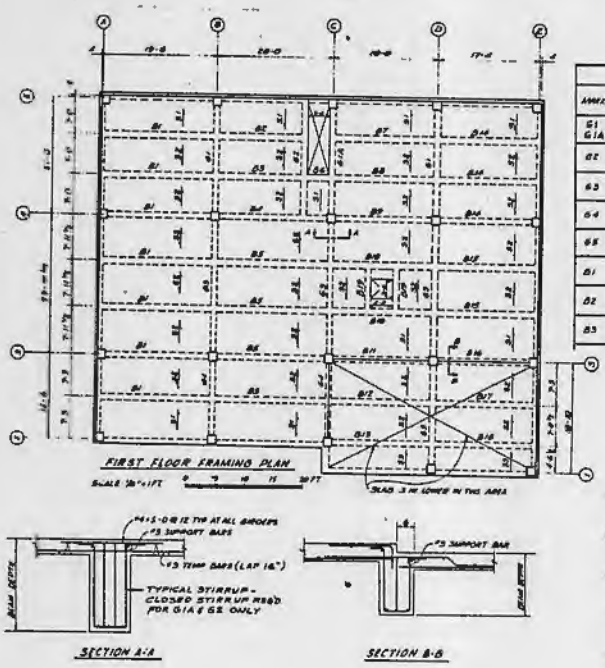
وبالله التوفيـــق ،،،



رسومات تنفيذية معمارية مرسومة بواسطة الحاسب الإكترونى .




تصميمات إنشائيه لاساسات وحوائط سانده مصممه ومرسومة بواسطة الحاسب الالكترونى .



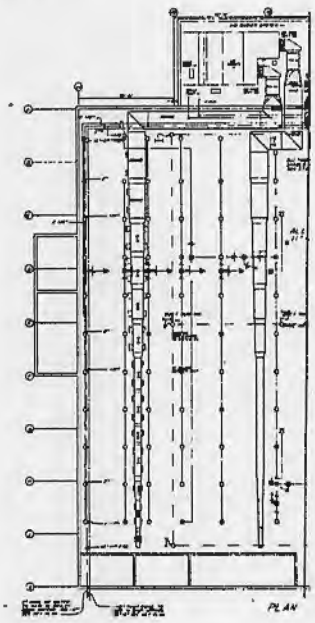

تصميمات انشائيه للأسقف مصممه ومرسومة ومكتوب جدوال تسليمها بواسطة الحاسب الإكترونى .

BEAM AND GIRDER SCHEDULE

| MARK | SIZE | LONGITUDINAL BOTTOM STRAIGHT | LONGITUDINAL TRUSS | LONGITUDINAL TOP | STIRRUPS MIN SIZE | STIRRUPS SPACING FROM FACE OF SUPPORT EACH END |
|---|---|---|---|---|---|---|
| G1 G1A | H+32 | 3-#8 | 3-#8 UNDER LANE | 1-#8 @ COLS B3, B2, C3 | #3 / #3 | 18@2, 3@6, 3@7, 3@9 & G1 / 18@2, 3@6, 11@9, G1A CLOSED STIRRUPS EVERYWHERE |
| G2 | 12+24 | 2-#8 | 2-#7 | 2-#9 1 1/2" EACH END | #3 | 18@2, 18@4, 3@5, 14@7 CLOSED STIRRUPS EVERYWHERE |
| G3 | 14+32 | 3-#7 | 2-#8 | 2-#11 1 1/2" EACH END | #3 | 18@2, 18@6, 18@7, 18@8, 18@12 3@13 |
| G4 | 12+22 | 2-#8 | 2-#6 | 1-#8 @ COLS D2, C2 | #3 | 18@1, 2@3, 2@4, 2@6, 18@8 3@9 |
| G5 | 14+30 | 3-#6 | 3-#8 | 1-#6 @ COL D1 | #3 | 18@1, 2@3, 4@4, 2@5, 18@7 12@11, 3@13 |
| G1 | 12+24 | 2-#9 | 1-#11 | 1-#9 @ COL LINE A | #3 | 18@2, 18@3, 18@6, 18@7, 18@9 4@10 |
| G2 | 12+20 | 2-#7 | 2-#8 | 2-#9 18 BOL LINE D 18 UNDER G2 | #3 | 18@2, 18@6, 18@7, 3@8 |
| G3 | 12+20 | 2-#7 | 2-#8 FITTED INTO G2 | 2-#9 (FITTED LINE D 18 UNDER G2 FITTED INTO G2) | #3 | 18@2, 18@6, 18@7, 3@8 |

ومع اتساع نطاق استخدام الحاسبات الالكترونية سوف يترتب عليه تطوير مواصفات التصميم والتنفيذ والتى تتناسب مع الامكانيات الكبيرة للحاسبات الالكترونية وكذلك سوف يترتب عليه ظهور نظريات جديدة والاهتمام بنظريات مستخدمة ولكن غير مألوفة للمهندس الممارس العادى ، وذلك نظرا لملائمة هذه النظريات لاستخدامات الحاسب الالكترونى وكذلك من المتوقع ان تتوارى بعض النظريات المألوفة الأخرى فى الظل وذلك لعدم سهولة برمجتها . وهذا التطوير والتغير سوف يكون له أكبر الأثر على نظم التعليم الهندسى العالمى أو المحلى .

وختاما أود أن أقول أنه منذ فترة ليست ببعيدة ترك المهندسون المساطر الحاسبة التى اعتادوا وتمرسوا عليها في الماضى واستخدموا الحاسبات الالكترونية وخصوصا أن بعضها أصبح في حجم الحاسبات اليدوية الشائع استخدامها الآن مع الفارق الكبير في المقدرة الحسابية ، حتى تستطيع أن نساير هذه « الثورة » في مجالات الهندسة المختلفة ونبدأ التفكير في حل المشكلات الهندسية بعقلية ونظم الحاسب الالكترونى .




جزء من مسقط يوضح أعمال تكييف الهواء مصممة ومرسومة بواسطة الحاسب الإكترونى .

تقليل الفاقد في المواد نتيجة عوامل عدم الوثوق مما يستدعى اضافة عامل كبير للأمان عند التصميم ، وهذا بدوره ينعكس على الانتاجية والتكلفة التنفيذية للمنشأ وخصوصا ان الاجزاء المصنعة المختلفة تنتج بكميات كبيرة والوفر في تكلفة انتاج وتنفيذ الوحدة الوحدة مهما كان بسيطا ، وسوف يكون تأثيره على التكلفة الاجمالية للمنشأ أو لمجموعة المنشئات كبيرا خصوصا إذا وضعنا في الاعتبار ان تصنيع البناء يرتبط أساسا بالانتاج المكثف من نفس الوحدة النمطية .

# برنـــامج للحاسب الالى لعمل جـــداول مســاحات وأوزان حـــديد التســـليـح

بقلـــم

الدكتور/ابراهيم محفوظ محمد ابراهيم
استاذ مساعد الهندسة الانشائية
بكلية الهندسة بشبرا

يمثل تحديد مساحات وأوزان حديد التسليح المستخدم فى الخرسانات مسألة أساسية سواء بالنسبة للمصمم بالمكتب أو المنفذ بالموقع أو لمعد تحليل العطاء لتقديم مناقصة لمشروع ما . ونسجل فى هذا المقال برنامجا مبسطا للحاسب الآلى مكتوبا بلغة الـ FORTRAN والخاص بعمل جداول لحساب مساحات وأوزان حديد التسليح لاقطارها المختلفة المستخدمة محليا أو عالميا ، بهدف اعطاء المهندس مقدمة لكيفية كتابة برنامج للحاسب الآلى فى أبسط صوره .

شكل رقم (١) يبين المسار البيانى FLOW CHART الخاص بالبرنامج المبين فى شكل رقم (٢) والخاص بحساب أوزان حديد التسليح للمتر الطولى لاقطار الحديد المختلفة ( ٦ر٨ حتى ٣٠ مم ) ولعدد أسياخ يتراوح من ١ الى ٥٠ والبرنامج ممكن تكرار تنفيذه عدة مرات على حسب اعداد الجداول المطلوبة فى حالة الرغبة فى طباعته آليا بواسطة الحاسب الآلى . ومن الملاحظ أن هذا البرنامج ممكن استخدامه لعمل جداول حسابات المساحات الاجمالية لحديد التسليح بعد تغيير العنوان فى سطر رقم ١٢ وعمل تعديل بحذف وزن وحدة الحجوم من السطر رقم ٢٣ . ويوضح الشكلان رقم (٣) ، (٤) الجداول الناتجة عن تنفيذ البرنامج .

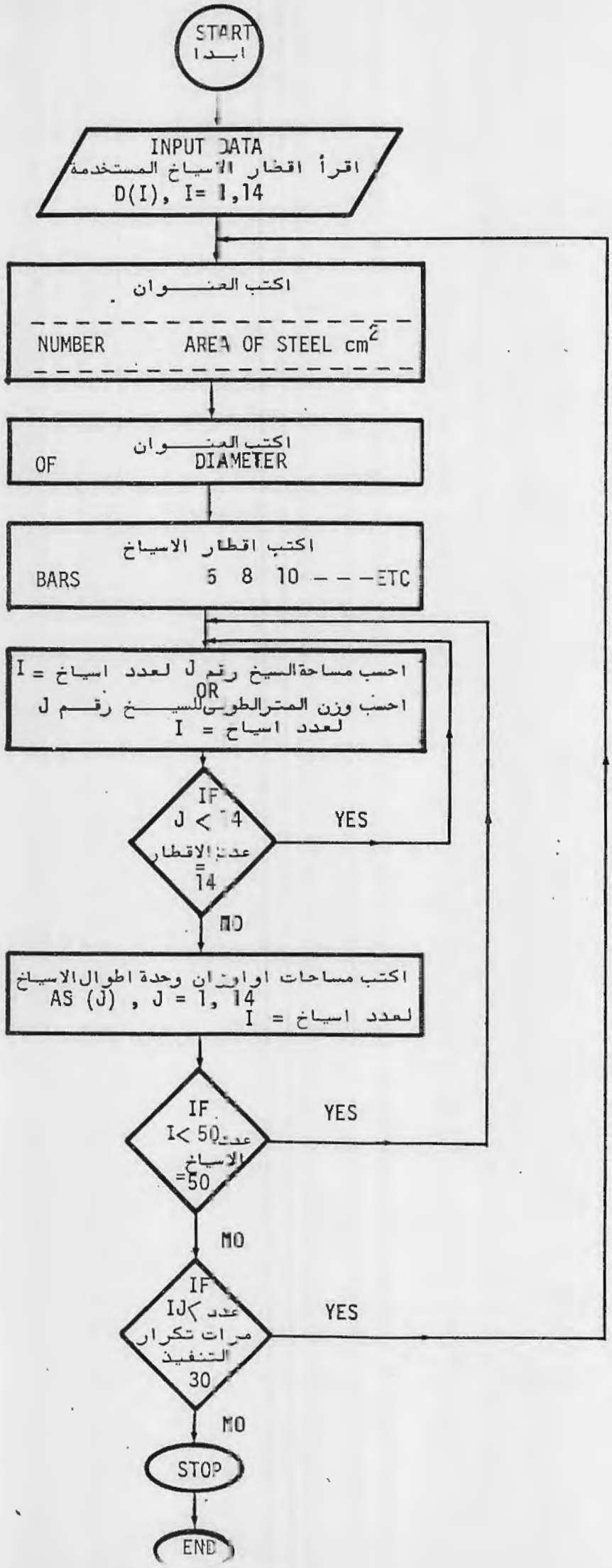



FIG 1 FLOW SHART.

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OPTIONS IN EFFECT:  NOLIST NOMAP NOXREF NOGOSTMT NODECK   SOURCE   TERM.  OBJECT FIXED  NOTEST
                    OPTIMIZE(0)  LANGLVL(77) NOFIPS   FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

                 *....*...1.........2.........3.........4.........5.........6.........7.*.......8

ISN        1          INTEGER D
ISN        2          DIMENSION D(14),AS(14)
ISN        3          READ(5,1) (D(I),I=1,14)
ISN        4     1    FORMAT (14I3)
ISN        5          DO 8 II=1,30
ISN        6          WRITE(6,10)
ISN        7     10   FORMAT (1H1)
ISN        8          WRITE (6,100)
ISN        9          WRITE (6,102)
ISN       10          WRITE(6,2)
ISN       11     102  FORMAT (/)
ISN       12     2    FORMAT (2X,'NUMBER',47X,'WEIGHT OF STEEL KG/M ',/)
ISN       13          WRITE (6,20)
ISN       14     20   FORMAT (8X,124(1H-))
ISN       15          WRITE(6,3)
ISN       16     3    FORMAT (2X,'OF',54X,'DIAMETER MM',/ )
ISN       17     100  FORMAT (1X,131(1H-))
ISN       18          WRITE(6,4) (D(I),I=1,14)
ISN       19     4    FORMAT (2X,'BARS',5X,14(I2,7X),/)
ISN       20          WRITE(6,100)
ISN       21          DO 7 I=1,50
ISN       22          DO 5 J=1,14
ISN       23     5    AS(J)=I*3.14159*D(J)**2*0.780/400.0
ISN       24          WRITE(6,6) I,(AS(JJ),JJ=1,14)
ISN       25     6    FORMAT (1X,I2,4X,14(F7.3,2X), )
ISN       26     7    CONTINUE
ISN       27     8    CONTINUE
ISN       28          STOP
ISN       29          END

*STATISTICS*   SOURCE STATEMENTS = 29, PROGRAM SIZE = 1146 BYTES, PROGRAM NAME = MAIN    PAGE:    1.

*STATISTICS*   NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 1 ******
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WEIGHT OF STEEL KG/M

| NO OF BARS | DIAMETER MM 6 | 8 | 10 | 12 | 13 | 14 | 16 | 18 | 19 | 20 | 22 | 25 | 28 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.221 | 0.392 | 0.613 | 0.882 | 1.035 | 1.201 | 1.568 | 1.985 | 2.212 | 2.450 | 2.965 | 3.829 | 4.803 | 5.513 |
| 2 | 0.441 | 0.784 | 1.225 | 1.764 | 2.071 | 2.401 | 3.137 | 3.970 | 4.423 | 4.901 | 5.930 | 7.658 | 9.606 | 11.027 |
| 3 | 0.662 | 1.176 | 1.838 | 2.646 | 3.106 | 3.602 | 4.705 | 5.955 | 6.635 | 7.351 | 8.895 | 11.486 | 14.409 | 16.540 |
| 4 | 0.882 | 1.568 | 2.450 | 3.529 | 4.141 | 4.803 | 6.273 | 7.939 | 8.846 | 9.802 | 11.860 | 15.315 | 19.211 | 22.054 |
| 5 | 1.103 | 1.960 | 3.063 | 4.411 | 5.177 | 6.004 | 7.841 | 9.924 | 11.058 | 12.252 | 14.825 | 19.144 | 24.014 | 27.567 |
| 6 | [illegible] | 2.352 | 3.676 | 5.293 | 6.212 | 7.204 | 9.410 | 11.909 | 13.269 | 14.703 | 17.790 | 22.973 | 28.817 | 33.081 |
| 7 | 1.5[illegible]4 | 2.744 | 4.288 | 6.175 | 7.247 | 8.405 | 10.978 | 13.894 | 15.481 | 17.153 | 20.755 | 26.802 | 33.620 | 38.594 |
| 8 | 1.7[illegible] | 3.137 | 4.901 | 7.057 | 8.282 | 9.606 | 12.546 | 15.879 | 17.692 | 19.604 | 23.720 | 30.630 | 38.423 | 44.108 |
| 9 | 1.9[illegible] | 3.529 | 5.513 | 7.939 | 9.318 | 10.806 | 14.115 | 17.864 | 19.904 | 22.054 | 26.685 | 34.459 | 43.226 | 49.621 |
| 10 | 2.2[illegible] | 3.921 | 6.126 | 8.822 | 10.353 | 12.007 | 15.683 | 19.849 | 22.115 | 24.504 | 29.650 | 38.288 | 48.029 | 55.135 |
| 11 | 2.426 | 4.313 | 6.739 | 9.704 | 11.388 | 13.208 | 17.251 | 21.833 | 24.327 | 26.955 | 32.615 | 42.117 | 52.831 | 60.648 |
| 12 | 2.646 | 4.705 | 7.351 | 10.586 | 12.424 | 14.409 | 18.819 | 23.818 | 26.538 | 29.405 | 35.580 | 45.946 | 57.634 | 66.162 |
| 13 | 2.867 | 5.097 | 7.964 | 11.468 | 13.459 | 15.609 | 20.388 | 25.803 | 28.750 | 31.856 | 38.545 | 49.775 | 62.437 | 71.675 |
| 14 | 3.088 | 5.489 | 8.577 | 12.350 | 14.494 | 16.810 | 21.956 | 27.788 | 30.961 | 34.306 | 41.510 | 53.603 | 67.240 | 77.189 |
| 15 | 3.308 | 5.881 | 9.189 | 13.232 | 15.530 | 18.011 | 23.524 | 29.773 | 33.173 | 36.757 | 44.475 | 57.432 | 72.043 | 82.702 |
| 16 | 3.529 | 6.273 | 9.802 | 14.115 | 16.565 | 19.211 | 25.092 | 31.758 | 35.384 | 39.207 | 47.441 | 61.261 | 76.846 | 88.216 |
| 17 | 3.749 | 6.665 | 10.414 | 14.997 | 17.600 | 20.412 | 26.661 | 33.743 | 37.596 | 41.657 | 50.406 | 65.090 | 81.649 | 93.729 |
| 18 | 3.97[illegible] | 7.057 | 11.027 | 15.879 | 18.636 | 21.613 | 28.229 | 35.727 | 39.807 | 44.108 | 53.371 | 68.919 | 86.452 | 99.243 |
| 19 | 4.190 | 7.449 | 11.640 | 16.761 | 19.671 | 22.814 | 29.797 | 37.712 | 42.019 | 46.558 | 56.336 | 72.747 | 91.254 | 104.756 |
| 20 | 4.411 | 7.841 | 12.252 | 17.643 | 20.706 | 24.014 | 31.366 | 39.697 | 44.230 | 49.009 | 59.301 | 76.576 | 96.057 | 110.270 |
| 21 | 4.631 | 8.233 | 12.865 | 18.525 | 21.742 | 25.215 | 32.934 | 41.682 | 46.442 | 51.459 | 62.266 | 80.405 | 100.860 | 115.783 |
| 22 | 4.852 | 8.626 | 13.477 | 19.407 | 22.777 | 26.416 | 34.502 | 43.667 | 48.653 | 53.910 | 65.231 | 84.234 | 105.663 | 121.297 |
| 23 | 5.072 | 9.018 | 14.090 | 20.290 | 23.812 | 27.616 | 36.070 | 45.652 | 50.865 | 56.360 | 68.196 | 88.063 | 110.466 | 126.810 |
| 24 | 5.293 | 9.410 | 14.703 | 21.172 | 24.847 | 28.817 | 37.639 | 47.637 | 53.077 | 58.811 | 71.161 | 91.891 | 115.269 | 132.324 |
| 25 | 5.513 | 9.802 | 15.315 | 22.054 | 25.883 | 30.018 | 39.207 | 49.621 | 55.288 | 61.261 | 74.126 | 95.720 | 120.072 | 137.837 |
| 26 | 5.[illegible]34 | 10.194 | 15.928 | 22.936 | 26.918 | 31.219 | 40.775 | 51.606 | 57.500 | 63.711 | 77.091 | 99.549 | 124.874 | 143.351 |
| 27 | 5.[illegible]5 | 10.586 | 16.540 | 23.818 | 27.953 | 32.419 | 42.344 | 53.591 | 59.711 | 66.162 | 80.056 | 103.378 | 129.677 | 148.864 |
| 28 | 6.1[illegible] | 10.978 | 17.153 | 24.700 | 28.989 | 33.620 | 43.912 | 55.576 | 61.923 | 68.612 | 83.021 | 107.207 | 134.480 | 154.378 |
| 29 | 6.39[illegible] | 11.370 | 17.766 | 25.583 | 30.024 | 34.821 | 45.480 | 57.561 | 64.134 | 71.063 | 85.986 | 111.036 | 139.283 | 159.891 |
| 30 | 6.616 | 11.762 | 18.378 | 26.465 | 31.059 | 36.021 | 47.048 | 59.546 | 66.346 | 73.513 | 88.951 | 114.864 | 144.086 | 165.405 |
| 31 | 6.837 | 12.154 | 18.991 | 27.347 | 32.095 | 37.222 | 48.617 | 61.531 | 68.557 | 75.964 | 91.916 | 118.693 | 148.889 | 170.918 |
| 32 | 7.057 | 12.546 | 19.604 | 28.229 | 33.130 | 38.423 | 50.185 | 63.515 | 70.769 | 78.414 | 94.881 | 122.522 | 153.692 | 176.432 |
| 33 | 7.278 | 12.938 | 20.216 | 29.111 | 34.165 | 39.624 | 51.753 | 65.500 | 72.980 | 80.865 | 97.846 | 126.351 | 158.494 | 181.945 |
| 34 | 7.498 | 13.330 | 20.829 | 29.993 | 35.201 | 40.824 | 53.322 | 67.485 | 75.192 | 83.315 | 100.811 | 130.180 | 163.297 | 187.459 |
| 35 | 7.719 | 13.722 | 21.441 | 30.876 | 36.236 | 42.025 | 54.890 | 69.470 | 77.403 | 85.765 | 103.776 | 134.008 | 168.100 | 192.972 |
| 36 | 7.939 | 14.115 | 22.054 | 31.758 | 37.271 | 43.226 | 56.458 | 71.455 | 79.615 | 88.216 | 106.741 | 137.837 | 172.903 | 198.485 |
| 37 | 8.160 | 14.507 | 22.667 | 32.640 | 38.306 | 44.426 | 58.026 | 73.440 | 81.826 | 90.666 | 109.706 | 141.666 | 177.706 | 203.999 |
| 38 | 8.380 | 14.899 | 23.279 | 33.522 | 39.342 | 45.627 | 59.595 | 75.425 | 84.038 | 93.117 | 112.671 | 145.495 | 182.509 | 209.512 |
| 39 | 8.601 | 15.291 | 23.892 | 34.404 | 40.377 | 46.828 | 61.163 | 77.409 | 86.249 | 95.567 | 115.636 | 149.324 | 187.312 | 215.026 |
| 40 | 8.822 | 15.683 | 24.504 | 35.286 | 41.412 | 48.029 | 62.731 | 79.394 | 88.461 | 98.018 | 118.601 | 153.152 | 192.114 | 220.539 |
| 41 | 9.042 | 16.075 | 25.117 | 36.168 | 42.448 | 49.229 | 64.300 | 81.379 | 90.672 | 100.468 | 121.566 | 156.981 | 196.917 | 226.053 |
| 42 | 9.263 | 16.467 | 25.730 | 37.051 | 43.483 | 50.430 | 65.868 | 83.364 | 92.884 | 102.918 | 124.531 | 160.810 | 201.720 | 231.566 |
| 43 | 9.483 | 16.859 | 26.342 | 37.933 | 44.518 | 51.631 | 67.436 | 85.349 | 95.095 | 105.369 | 127.496 | 164.639 | 206.523 | 237.080 |
| 44 | 9.704 | 17.251 | 26.955 | 38.815 | 45.554 | 52.831 | 69.004 | 87.334 | 97.307 | 107.819 | 130.461 | 168.468 | 211.326 | 242.593 |
| 45 | 9.924 | 17.643 | 27.567 | 39.697 | 46.589 | 54.032 | 70.573 | 89.319 | 99.518 | 110.270 | 133.426 | 172.296 | 216.129 | 248.1[illegible] |
| 46 | 10.145 | 18.035 | 28.180 | 40.579 | 47.624 | 55.233 | 72.141 | 91.303 | 101.730 | 112.720 | 136.391 | 176.125 | 220.932 | 25[illegible] |
| 47 | 10.365 | 18.427 | 28.793 | 41.461 | 48.660 | 56.434 | 73.709 | 93.288 | 103.942 | 115.171 | 139.356 | 179.954 | 225.734 | 25[illegible] |
| 48 | 10.586 | 18.819 | 29.405 | 42.344 | 49.695 | 57.634 | 75.278 | 95.273 | 106.153 | 117.621 | 142.321 | 183.783 | 230.537 | 2[illegible].7 |
| 49 | 10.806 | 19.211 | 30.018 | 43.226 | 50.730 | 58.835 | 76.846 | 97.258 | 108.365 | 120.072 | 145.287 | 187.612 | 235.340 | [illegible].61 |
| 50 | 11.027 | 19.604 | 30.630 | 44.108 | 51.766 | 60.036 | 78.414 | 99.243 | 110.576 | 122.522 | 148.252 | 191.440 | 240.143 | [illegible].674 |

# COMPUTER AIDED DEVELOPMENT OF DESIGN AID TABLES FOR REINFORCED CONCRETE STRUCTURES

BY

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The objective of this work is to present a fortran computer program for the development of design aid tables for reinforced concrete sections subjected to bending moments. The tables can be used to greatly simplify the design of a variety of reinforced concrete structural elements such as, beams having rectangular, T or L sections, slabs, flat slabs, footings ... etc... The computer program is based on the " ULTIMATE " strength design method of the ACI 318-83 building code requirements for reinforced concrete.

In the " ULTIMATE " strength design method, the depth of the reinforced concrete section, d can be expressed in terms of the " ULTIMATE MOMENT " $M_u$ and the width, b as follows:

$$d = \sqrt{\frac{M_u}{\phi B \quad R_n}} \qquad (1)$$

where

$$R_n = F_y [\, \rho - \rho^2 \; F_y / [\, 1.7 \quad F_c' ]\, ] \qquad (2)$$

and $\rho$ is the percentage of steel $= A_s / bd$ (2b)

$F_y$ = Yield strength of the reinforcing steel

$F_c'$ = Compressive strength of concrete cylinder after 28 days.

It is pointed out that the design aid tables give the values of $R_n$ for various percentages of steel, $\rho$ , for different values of $F_c'$ and $F_y$. In the preceding Equation, 1 , $\phi$ is the strength reduction factor and is equal to 0.9 for flexure. Note that the strength reduction factor accounts for uncertainties in design computations and relative importance of various types of members, and provides for the possiblity that small address variations in material strengths, workmanship, and dimensions, while individually within acceptable tolerances and limits, may conbine to result in understrength.

The required moment strength, $M_u$ or, the factored moment, is computed as:-

$$M_u = LF_{DL} \quad M_d + LF_{LL} \quad M_L \tag{3}$$

where, $M_d$ and $M_L$ are the moments due to service dead and live loads, respectively, and, $LF_{DL}$ and $LF_{LL}$ are the load factors for dead loads and live loads, which provide for excess load effects from such possible sources as overloads and simplified assumptions in structural analysis. Recommended load factors for the middle eastern region can be taken equal to, $LF_{DL} = 1.5$ and $LF_{LL} = 1.8$

The flow chart of the computer program and the listing of the program are shown in figures 1 and 2, respectively. Sample output design aid tables are presented in the tables, for various combinations of $F'_c$ and $F_y$. It should be noted that the program is independent of the qualities and units of both concrete and steel and can be used in the cases where the quality of concrete is represented by its cube strength.

1...5...10...15...20...25...30...35...40...45...50...55...60...65...70...75...80 .....COMMENTS

HEAD

TABLES FOR THE DESIGN OF R.C.
SECTIONS SUBJECTED TO BENDING
MOMENT

D = SQRT ( MULT / Q RN B )

ROW = AS / ( B D )

WHERE Q = 0.9

MULT = ULTIMATE MOMENT

D = DEPTH OF THE BEAM

B = WIDTH OF THE BEAM

AS = AREA OF STEEL

FY = YIELD STRENGTH OF STEEL

FC1 = COMPRESSIVE STRENGTH OF CONCRETE
CYLINDER AFTER 28 DAYS

RN = REFER TO TABLES

PREPARED BY DR.IBRAHIM MAHFOUZ MOHAMED IBRAHIM

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OPTIONS IN EFFECT:  NOLIST NOMAP NOXREF NOGOSTMT NODECK   SOURCE   TERM   OBJECT FIXED  NOTEST
                    OPTIMIZE(0)  LANGLVL(77) NOFIPS   FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

                 *....*...1.........2.........3.........4.........5.........6.........7.*.......8

ISN        1          DIMENSION ROW(500),RN(500)
ISN        2          READ(5,1) FC1,FY
ISN        3    1     FORMAT(3F10.4)
ISN        4          READ(5,2) N
ISN        5    2     FORMAT(1I3)
ISN        6          ROW(1)=0.0000
ISN        7          DO 3 I=1,N
ISN        8          RN(I)=ROW(I)*FY*(1.0-ROW(I)*FY/(1.7*FC1))
ISN        9          J=I+1
ISN       10    3     ROW(J)=ROW(I)+0.0001
ISN       11          WRITE(6,9)
ISN       12    9     FORMAT(1H1,35X,'*******************************************')
ISN       13    4     FORMAT(35X,'*******************************************')
ISN       14          WRITE(6,5) FC1,FY
ISN       15    5     FORMAT(40X,4HFC1=F10.4,5X,3HFY=F10.4)
ISN       16          WRITE(6,4)
ISN       17          WRITE(6,6)
ISN       18    6     FORMAT(7X,6(3HROW,7X,2HRN,7X))
ISN       19          DO 7 I=1,60
ISN       20          II=I+60
ISN       21          JJ=II+60
ISN       22          KK=JJ+60
ISN       23          IK=KK+60
ISN       24          IJ=IK+60
ISN       25    7     WRITE(6,8) ROW(I),RN(I),ROW(II),RN(II),ROW(JJ),RN(JJ),ROW(KK),
                     X RN(KK),ROW(IK),RN(IK),ROW(IJ),RN(IJ)
ISN       26    8     FORMAT(5X,6(F7.4,2X,F7.4,3X))
ISN       27          STOP
ISN       28          END

*STATISTICS*  SOURCE STATEMENTS = 28, PROGRAM SIZE = 5162 BYTES, PROGRAM NAME = MAIN    PAGE:    1.

*STATISTICS*   NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 1 ******
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FC1= 200.0000 FY= 2300.0000

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| ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.0060 | 13.2398 | 0.0120 | 25.3592 | 0.0180 | 36.3584 | 0.0240 | 46.2375 | 0.0300 | 54.9963 |
| 0.0001 | 0.2298 | 0.0061 | 13.4510 | 0.0121 | 25.5517 | 0.0181 | 36.5323 | 0.0241 | 46.3926 | 0.0301 | 55.1328 |
| 0.0002 | 0.4594 | 0.0062 | 13.6618 | 0.0122 | 25.7439 | 0.0182 | 36.7058 | 0.0242 | 46.5475 | 0.0302 | 55.2690 |
| 0.0003 | 0.6886 | 0.0063 | 13.8724 | 0.0123 | 25.9357 | 0.0183 | 36.8790 | 0.0243 | 46.7020 | 0.0303 | 55.4048 |
| 0.0004 | 0.9175 | 0.0064 | 14.0826 | 0.0124 | 26.1273 | 0.0184 | 37.0519 | 0.0244 | 46.8562 | 0.0304 | 55.5404 |
| 0.0005 | 1.1461 | 0.0065 | 14.2925 | 0.0125 | 26.3186 | 0.0185 | 37.2245 | 0.0245 | 47.0101 | 0.0305 | 55.6756 |
| 0.0006 | 1.3744 | 0.0066 | 14.5021 | 0.0126 | 26.5095 | 0.0186 | 37.3967 | 0.0246 | 47.1637 | 0.0306 | 55.8106 |
| 0.0007 | 1.6024 | 0.0067 | 14.7114 | 0.0127 | 26.7001 | 0.0187 | 37.5687 | 0.0247 | 47.3170 | 0.0307 | 55.9452 |
| 0.0008 | 1.8300 | 0.0068 | 14.9204 | 0.0128 | 26.8905 | 0.0188 | 37.7403 | 0.0248 | 47.4700 | 0.0308 | 56.0795 |
| 0.0009 | 2.0574 | 0.0069 | 15.1291 | 0.0129 | 27.0805 | 0.0189 | 37.9117 | 0.0249 | 47.6227 | 0.0309 | 56.2135 |
| 0.0010 | 2.2844 | 0.0070 | 15.3375 | 0.0130 | 27.2702 | 0.0190 | 38.0827 | 0.0250 | 47.7751 | 0.0310 | 56.3472 |
| 0.0011 | 2.5112 | 0.0071 | 15.5455 | 0.0131 | 27.4596 | 0.0191 | 38.2534 | 0.0251 | 47.9271 | 0.0311 | 56.4806 |
| 0.0012 | 2.7376 | 0.0072 | 15.7533 | 0.0132 | 27.6487 | 0.0192 | 38.4238 | 0.0252 | 48.0788 | 0.0312 | 56.6137 |
| 0.0013 | 2.9637 | 0.0073 | 15.9607 | 0.0133 | 27.8374 | 0.0193 | 38.5939 | 0.0253 | 48.2303 | 0.0313 | 56.7464 |
| 0.0014 | 3.1895 | 0.0074 | 16.1678 | 0.0134 | 28.0259 | 0.0194 | 38.7637 | 0.0254 | 48.3814 | 0.0314 | 56.8789 |
| 0.0015 | 3.4150 | 0.0075 | 16.3746 | 0.0135 | 28.2140 | 0.0195 | 38.9332 | 0.0255 | 48.5322 | 0.0315 | 57.0110 |
| 0.0016 | 3.6402 | 0.0076 | 16.5811 | 0.0136 | 28.4018 | 0.0196 | 39.1024 | 0.0256 | 48.6827 | 0.0316 | 57.1428 |
| 0.0017 | 3.8650 | 0.0077 | 16.7873 | 0.0137 | 28.5894 | 0.0197 | 39.2712 | 0.0257 | 48.8329 | 0.0317 | 57.2743 |
| 0.0018 | 4.0896 | 0.0078 | 16.9932 | 0.0138 | 28.7766 | 0.0198 | 39.4398 | 0.0258 | 48.9827 | 0.0318 | 57.4055 |
| 0.0019 | 4.3138 | 0.0079 | 17.1988 | 0.0139 | 28.9635 | 0.0199 | 39.6080 | 0.0259 | 49.1323 | 0.0319 | 57.5364 |
| 0.0020 | 4.5378 | 0.0080 | 17.4040 | 0.0140 | 29.1501 | 0.0200 | 39.7759 | 0.0260 | 49.2815 | 0.0320 | 57.6670 |
| 0.0021 | 4.7614 | 0.0081 | 17.6090 | 0.0141 | 29.3363 | 0.0201 | 39.9435 | 0.0261 | 49.4305 | 0.0321 | 57.7973 |
| 0.0022 | 4.9847 | 0.0082 | 17.8136 | 0.0142 | 29.5223 | 0.0202 | 40.1108 | 0.0262 | 49.5791 | 0.0322 | 57.9272 |
| 0.0023 | 5.2077 | 0.0083 | 18.0179 | 0.0143 | 29.7080 | 0.0203 | 40.2778 | 0.0263 | 49.7274 | 0.0323 | 58.0569 |
| 0.0024 | 5.4304 | 0.0084 | 18.2220 | 0.0144 | 29.8933 | 0.0204 | 40.4445 | 0.0264 | 49.8754 | 0.0324 | 58.1862 |
| 0.0025 | 5.6527 | 0.0085 | 18.4257 | 0.0145 | 30.0783 | 0.0205 | 40.6108 | 0.0265 | 50.0231 | 0.0325 | 58.3152 |
| 0.0026 | 5.8748 | 0.0086 | 18.6290 | 0.0146 | 30.2631 | 0.0206 | 40.7769 | 0.0266 | 50.1705 | 0.0326 | 58.4439 |
| 0.0027 | 6.0966 | 0.0087 | 18.8321 | 0.0147 | 30.4475 | 0.0207 | 40.9426 | 0.0267 | 50.3176 | 0.0327 | 58.5723 |
| 0.0028 | 6.3180 | 0.0088 | 19.0349 | 0.0148 | 30.6316 | 0.0208 | 41.1080 | 0.0268 | 50.4643 | 0.0328 | 58.7004 |
| 0.0029 | 6.5391 | 0.0089 | 19.2374 | 0.0149 | 30.8154 | 0.0209 | 41.2731 | 0.0269 | 50.6108 | 0.0329 | 58.8282 |
| 0.0030 | 6.7600 | 0.0090 | 19.4395 | 0.0150 | 30.9988 | 0.0210 | 41.4380 | 0.0270 | 50.7569 | 0.0330 | 58.9557 |
| 0.0031 | 6.9805 | 0.0091 | 19.6413 | 0.0151 | 31.1820 | 0.0211 | 41.6025 | 0.0271 | 50.9027 | 0.0331 | 59.0828 |
| 0.0032 | 7.2007 | 0.0092 | 19.8428 | 0.0152 | 31.3649 | 0.0212 | 41.7666 | 0.0272 | 51.0482 | 0.0332 | 59.2097 |
| 0.0033 | 7.4206 | 0.0093 | 20.0441 | 0.0153 | 31.5474 | 0.0213 | 41.9305 | 0.0273 | 51.1935 | 0.0333 | 59.3362 |
| 0.0034 | 7.6401 | 0.0094 | 20.2450 | 0.0154 | 31.7296 | 0.0214 | 42.0941 | 0.0274 | 51.3383 | 0.0334 | 59.4624 |
| 0.0035 | 7.8594 | 0.0095 | 20.4456 | 0.0155 | 31.9115 | 0.0215 | 42.2573 | 0.0275 | 51.4829 | 0.0335 | 59.5883 |
| 0.0036 | 8.0783 | 0.0096 | 20.6458 | 0.0156 | 32.0932 | 0.0216 | 42.4203 | 0.0276 | 51.6272 | 0.0336 | 59.7139 |
| 0.0037 | 8.2970 | 0.0097 | 20.8458 | 0.0157 | 32.2744 | 0.0217 | 42.5829 | 0.0277 | 51.7711 | 0.0337 | 59.8392 |
| 0.0038 | 8.5153 | 0.0098 | 21.0455 | 0.0158 | 32.4554 | 0.0218 | 42.7452 | 0.0278 | 51.9148 | 0.0338 | 59.9642 |
| 0.0039 | 8.7333 | 0.0099 | 21.2448 | 0.0159 | 32.6361 | 0.0219 | 42.9072 | 0.0279 | 52.0581 | 0.0339 | 60.0889 |
| 0.0040 | 8.9510 | 0.0100 | 21.4438 | 0.0160 | 32.8165 | 0.0220 | 43.0689 | 0.0280 | 52.2012 | 0.0340 | 60.2132 |
| 0.0041 | 9.1684 | 0.0101 | 21.6426 | 0.0161 | 32.9965 | 0.0221 | 43.2303 | 0.0281 | 52.3439 | 0.0341 | 60.3372 |
| 0.0042 | 9.3855 | 0.0102 | 21.8410 | 0.0162 | 33.1763 | 0.0222 | 43.3914 | 0.0282 | 52.4863 | 0.0342 | 60.4610 |
| 0.0043 | 9.6023 | 0.0103 | 22.0391 | 0.0163 | 33.3557 | 0.0223 | 43.5521 | 0.0283 | 52.6284 | 0.0343 | 60.5844 |
| 0.0044 | 9.8187 | 0.0104 | 22.2369 | 0.0164 | 33.5348 | 0.0224 | 43.7126 | 0.0284 | 52.7701 | 0.0344 | 60.7075 |
| 0.0045 | 10.0349 | 0.0105 | 22.4344 | 0.0165 | 33.7136 | 0.0225 | 43.8727 | 0.0285 | 52.9116 | 0.0345 | 60.8303 |
| 0.0046 | 10.2507 | 0.0106 | 22.6315 | 0.0166 | 33.8921 | 0.0226 | 44.0325 | 0.0286 | 53.0528 | 0.0346 | 60.9528 |
| 0.0047 | 10.4663 | 0.0107 | 22.8284 | 0.0167 | 34.0703 | 0.0227 | 44.1921 | 0.0287 | 53.1936 | 0.0347 | 61.0750 |
| 0.0048 | 10.6815 | 0.0108 | 23.0249 | 0.0168 | 34.2482 | 0.0228 | 44.3513 | 0.0288 | 53.3342 | 0.0348 | 61.1969 |
| 0.0049 | 10.8964 | 0.0109 | 23.2212 | 0.0169 | 34.4258 | 0.0229 | 44.5102 | 0.0289 | 53.4744 | 0.0349 | 61.3184 |
| 0.0050 | 11.1110 | 0.0110 | 23.4171 | 0.0170 | 34.6030 | 0.0230 | 44.6687 | 0.0290 | 53.6143 | 0.0350 | 61.4397 |
| 0.0051 | 11.3253 | 0.0111 | 23.6127 | 0.0171 | 34.7799 | 0.0231 | 44.8270 | 0.0291 | 53.7539 | 0.0351 | 61.5606 |
| 0.0052 | 11.5392 | 0.0112 | 23.8080 | 0.0172 | 34.9566 | 0.0232 | 44.9850 | 0.0292 | 53.8932 | 0.0352 | 61.6812 |
| 0.0053 | 11.7529 | 0.0113 | 24.0030 | 0.0173 | 35.1329 | 0.0233 | 45.1426 | 0.0293 | 54.0322 | 0.0353 | 61.8015 |
| 0.0054 | 11.9662 | 0.0114 | 24.1977 | 0.0174 | 35.3089 | 0.0234 | 45.3000 | 0.0294 | 54.1708 | 0.0354 | 61.9215 |
| 0.0055 | 12.1793 | 0.0115 | 24.3920 | 0.0175 | 35.4846 | 0.0235 | 45.4570 | 0.0295 | 54.3092 | 0.0355 | 62.0412 |
| 0.0056 | 12.3920 | 0.0116 | 24.5861 | 0.0176 | 35.6600 | 0.0236 | 45.6137 | 0.0296 | 54.4472 | 0.0356 | 62.1606 |
| 0.0057 | 12.6044 | 0.0117 | 24.7798 | 0.0177 | 35.8351 | 0.0237 | 45.7701 | 0.0297 | 54.5850 | 0.0357 | 62.2796 |
| 0.0058 | 12.8165 | 0.0118 | 24.9733 | 0.0178 | 36.0098 | 0.0238 | 45.9262 | 0.0298 | 54.7224 | 0.0358 | 62.3984 |
| 0.0059 | 13.0283 | 0.0119 | 25.1664 | 0.0179 | 36.1843 | 0.0239 | 46.0820 | 0.0299 | 54.8595 | 0.0359 | 62.5168 |

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FC1= 180.0000 FY= 3600.0000
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| ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.0060 | 20.0751 | 0.0120 | 37.1007 | 0.0180 | 51.0770 | 0.0240 | 62.0040 | 0.0300 | 69.8817 |
| 0.0001 | 0.3596 | 0.0061 | 20.3839 | 0.0121 | 37.3586 | 0.0181 | 51.2841 | 0.0241 | 62.1603 | 0.0301 | 69.9872 |
| 0.0002 | 0.7183 | 0.0062 | 20.6918 | 0.0122 | 37.6157 | 0.0182 | 51.4904 | 0.0242 | 62.3158 | 0.0302 | 70.0918 |
| 0.0003 | 1.0762 | 0.0063 | 20.9988 | 0.0123 | 37.8720 | 0.0183 | 51.6958 | 0.0243 | 62.4703 | 0.0303 | 70.1956 |
| 0.0004 | 1.4332 | 0.0064 | 21.3050 | 0.0124 | 38.1273 | 0.0184 | 51.9003 | 0.0244 | 62.6241 | 0.0304 | 70.2985 |
| 0.0005 | 1.7894 | 0.0065 | 21.6104 | 0.0125 | 38.3819 | 0.0185 | 52.1041 | 0.0245 | 62.7770 | 0.0305 | 70.4006 |
| 0.0006 | 2.1448 | 0.0066 | 21.9149 | 0.0126 | 38.6356 | 0.0186 | 52.3069 | 0.0246 | 62.9290 | 0.0306 | 70.5018 |
| 0.0007 | 2.4993 | 0.0067 | 22.2186 | 0.0127 | 38.8884 | 0.0187 | 52.5090 | 0.0247 | 63.0802 | 0.0307 | 70.6022 |
| 0.0008 | 2.8529 | 0.0068 | 22.5214 | 0.0128 | 39.1404 | 0.0188 | 52.7101 | 0.0248 | 63.2306 | 0.0308 | 70.7017 |
| 0.0009 | 3.2057 | 0.0069 | 22.8234 | 0.0129 | 39.3916 | 0.0189 | 52.9105 | 0.0249 | 63.3801 | 0.0309 | 70.8004 |
| 0.0010 | 3.5576 | 0.0070 | 23.1245 | 0.0130 | 39.6419 | 0.0190 | 53.1099 | 0.0250 | 63.5287 | 0.0310 | 70.8982 |
| 0.0011 | 3.9087 | 0.0071 | 23.4248 | 0.0131 | 39.8913 | 0.0191 | 53.3086 | 0.0251 | 63.6765 | 0.0311 | 70.9952 |
| 0.0012 | 4.2590 | 0.0072 | 23.7242 | 0.0132 | 40.1399 | 0.0192 | 53.5064 | 0.0252 | 63.8235 | 0.0312 | 71.0914 |
| 0.0013 | 4.6084 | 0.0073 | 24.0228 | 0.0133 | 40.3877 | 0.0193 | 53.7033 | 0.0253 | 63.9696 | 0.0313 | 71.1867 |
| 0.0014 | 4.9570 | 0.0074 | 24.3205 | 0.0134 | 40.6346 | 0.0194 | 53.8994 | 0.0254 | 64.1149 | 0.0314 | 71.2811 |
| 0.0015 | 5.3047 | 0.0075 | 24.6174 | 0.0135 | 40.8807 | 0.0195 | 54.0947 | 0.0255 | 64.2593 | 0.0315 | 71.3747 |
| 0.0016 | 5.6516 | 0.0076 | 24.9135 | 0.0136 | 41.1259 | 0.0196 | 54.2890 | 0.0256 | 64.4029 | 0.0316 | 71.4675 |
| 0.0017 | 5.9976 | 0.0077 | 25.2086 | 0.0137 | 41.3703 | 0.0197 | 54.4826 | 0.0257 | 64.5456 | 0.0317 | 71.5594 |
| 0.0018 | 6.3428 | 0.0078 | 25.5030 | 0.0138 | 41.6138 | 0.0198 | 54.6753 | 0.0258 | 64.6875 | 0.0318 | 71.6505 |
| 0.0019 | 6.6871 | 0.0079 | 25.7965 | 0.0139 | 41.8565 | 0.0199 | 54.8672 | 0.0259 | 64.8286 | 0.0319 | 71.7407 |
| 0.0020 | 7.0306 | 0.0080 | 26.0891 | 0.0140 | 42.0983 | 0.0200 | 55.0582 | 0.0260 | 64.9688 | 0.0320 | 71.8300 |
| 0.0021 | 7.3732 | 0.0081 | 26.3810 | 0.0141 | 42.3393 | 0.0201 | 55.2483 | 0.0261 | 65.1081 | 0.0321 | 71.9185 |
| 0.0022 | 7.7150 | 0.0082 | 26.6719 | 0.0142 | 42.5794 | 0.0202 | 55.4377 | 0.0262 | 65.2466 | 0.0322 | 72.0062 |
| 0.0023 | 8.0559 | 0.0083 | 26.9620 | 0.0143 | 42.8187 | 0.0203 | 55.6261 | 0.0263 | 65.3842 | 0.0323 | 72.0931 |
| 0.0024 | 8.3960 | 0.0084 | 27.2513 | 0.0144 | 43.0572 | 0.0204 | 55.8137 | 0.0264 | 65.5210 | 0.0324 | 72.1790 |
| S 0.0025 | 8.7353 | 0.0085 | 27.5397 | 0.0145 | 43.2948 | 0.0205 | 56.0005 | 0.0265 | 65.6570 | 0.0325 | 72.2642 |
| 0.0026 | 9.0737 | 0.0086 | 27.8273 | 0.0146 | 43.5315 | 0.0206 | 56.1864 | 0.0266 | 65.7921 | 0.0326 | 72.3484 |
| 0.0027 | 9.4112 | 0.0087 | 28.1140 | 0.0147 | 43.7674 | 0.0207 | 56.3715 | 0.0267 | 65.9263 | 0.0327 | 72.4319 |
| 0.0028 | 9.7479 | 0.0088 | 28.3999 | 0.0148 | 44.0025 | 0.0208 | 56.5558 | 0.0268 | 66.0598 | 0.0328 | 72.5145 |
| 0.0029 | 10.0838 | 0.0089 | 28.6849 | 0.0149 | 44.2367 | 0.0209 | 56.7392 | 0.0269 | 66.1923 | 0.0329 | 72.5962 |
| 0.0030 | 10.4188 | 0.0090 | 28.9691 | 0.0150 | 44.4700 | 0.0210 | 56.9217 | 0.0270 | 66.3241 | 0.0330 | 72.6771 |
| 0.0031 | 10.7530 | 0.0091 | 29.2524 | 0.0151 | 44.7025 | 0.0211 | 57.1034 | 0.0271 | 66.4549 | 0.0331 | 72.7572 |
| 0.0032 | 11.0863 | 0.0092 | 29.5349 | 0.0152 | 44.9342 | 0.0212 | 57.2842 | 0.0272 | 66.5849 | 0.0332 | 72.8364 |
| 0.0033 | 11.4188 | 0.0093 | 29.8166 | 0.0153 | 45.1650 | 0.0213 | 57.4642 | 0.0273 | 66.7141 | 0.0333 | 72.9147 |
| 0.0034 | 11.7504 | 0.0094 | 30.0974 | 0.0154 | 45.3950 | 0.0214 | 57.6434 | 0.0274 | 66.8425 | 0.0334 | 72.9922 |
| 0.0035 | 12.0812 | 0.0095 | 30.3773 | 0.0155 | 45.6241 | 0.0215 | 57.8217 | 0.0275 | 66.9699 | 0.0335 | 73.0689 |
| 0.0036 | 12.4111 | 0.0096 | 30.6564 | 0.0156 | 45.8524 | 0.0216 | 57.9991 | 0.0276 | 67.0966 | 0.0336 | 73.1447 |
| 0.0037 | 12.7402 | 0.0097 | 30.9347 | 0.0157 | 46.0798 | 0.0217 | 58.1758 | 0.0277 | 67.2224 | 0.0337 | 73.2197 |
| 0.0038 | 13.0684 | 0.0098 | 31.2121 | 0.0158 | 46.3064 | 0.0218 | 58.3515 | 0.0278 | 67.3473 | 0.0338 | 73.2938 |
| b 0.0039 | 13.3958 | 0.0099 | 31.4886 | 0.0159 | 46.5322 | 0.0219 | 58.5264 | 0.0279 | 67.4714 | 0.0339 | 73.3671 |
| 0.0040 | 13.7223 | 0.0100 | 31.7643 | 0.0160 | 46.7571 | 0.0220 | 58.7005 | 0.0280 | 67.5947 | 0.0340 | 73.4395 |
| 0.0041 | 14.0480 | 0.0101 | 32.0392 | 0.0161 | 46.9811 | 0.0221 | 58.8737 | 0.0281 | 67.7171 | 0.0341 | 73.5111 |
| 0.0042 | 14.3729 | 0.0102 | 32.3132 | 0.0162 | 47.2043 | 0.0222 | 59.0461 | 0.0282 | 67.8386 | 0.0342 | 73.5818 |
| 0.0043 | 14.6969 | 0.0103 | 32.5864 | 0.0163 | 47.4267 | 0.0223 | 59.2176 | 0.0283 | 67.9593 | 0.0343 | 73.6517 |
| 0.0044 | 15.0200 | 0.0104 | 32.8587 | 0.0164 | 47.6482 | 0.0224 | 59.3883 | 0.0284 | 68.0792 | 0.0344 | 73.7207 |
| 0.0045 | 15.3423 | 0.0105 | 33.1302 | 0.0165 | 47.8688 | 0.0225 | 59.5582 | 0.0285 | 68.1982 | 0.0345 | 73.7889 |
| 0.0046 | 15.6637 | 0.0106 | 33.4008 | 0.0166 | 48.0886 | 0.0226 | 59.7271 | 0.0286 | 68.3164 | 0.0346 | 73.8563 |
| 0.0047 | 15.9844 | 0.0107 | 33.6706 | 0.0167 | 48.3076 | 0.0227 | 59.8953 | 0.0287 | 68.4337 | 0.0347 | 73.9228 |
| 0.0048 | 16.3041 | 0.0108 | 33.9395 | 0.0168 | 48.5257 | 0.0228 | 60.0626 | 0.0288 | 68.5501 | 0.0348 | 73.9884 |
| 0.0049 | 16.6230 | 0.0109 | 34.2076 | 0.0169 | 48.7430 | 0.0229 | 60.2290 | 0.0289 | 68.6658 | 0.0349 | 74.0532 |
| 0.0050 | 16.9411 | 0.0110 | 34.4749 | 0.0170 | 48.9594 | 0.0230 | 60.3946 | 0.0290 | 68.7805 | 0.0350 | 74.1172 |
| 0.0051 | 17.2583 | 0.0111 | 34.7413 | 0.0171 | 49.1750 | 0.0231 | 60.5594 | 0.0291 | 68.8945 | 0.0351 | 74.1803 |
| 0.0052 | 17.5747 | 0.0112 | 35.0068 | 0.0172 | 49.3897 | 0.0232 | 60.7233 | 0.0292 | 69.0076 | 0.0352 | 74.2426 |
| 0.0053 | 17.8902 | 0.0113 | 35.2715 | 0.0173 | 49.6036 | 0.0233 | 60.8863 | 0.0293 | 69.1198 | 0.0353 | 74.3040 |
| 0.0054 | 18.2049 | 0.0114 | 35.5354 | 0.0174 | 49.8166 | 0.0234 | 61.0486 | 0.0294 | 69.2312 | 0.0354 | 74.3645 |
| 0.0055 | 18.5187 | 0.0115 | 35.7984 | 0.0175 | 50.0288 | 0.0235 | 61.2099 | 0.0295 | 69.3417 | 0.0355 | 74.4243 |
| 0.0056 | 18.8317 | 0.0116 | 36.0605 | 0.0176 | 50.2401 | 0.0236 | 61.3704 | 0.0296 | 69.4514 | 0.0356 | 74.4832 |
| 0.0057 | 19.1438 | 0.0117 | 36.3219 | 0.0177 | 50.4506 | 0.0237 | 61.5301 | 0.0297 | 69.5603 | 0.0357 | 74.5412 |
| 0.0058 | 19.4551 | 0.0118 | 36.5823 | 0.0178 | 50.6603 | 0.0238 | 61.6889 | 0.0298 | 69.6683 | 0.0358 | 74.5984 |
| 0.0059 | 19.7655 | 0.0119 | 36.8419 | 0.0179 | 50.8691 | 0.0239 | 61.8469 | 0.0299 | 69.7754 | 0.0359 | 74.6547 |

0.0295 0.0604

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FC1= 200.0000 FY= 3600.0000

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| ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.0060 | 20.2276 | 0.0120 | 37.7106 | 0.0180 | 52.4492 | 0.0240 | 64.4435 | 0.0300 | 73.6934 |
| 0.0001 | 0.3596 | 0.0061 | 20.5415 | 0.0121 | 37.9787 | 0.0181 | 52.6716 | 0.0241 | 64.6201 | 0.0301 | 73.8243 |
| 0.0002 | 0.7185 | 0.0062 | 20.8546 | 0.0122 | 38.2461 | 0.0182 | 52.8932 | 0.0242 | 64.7960 | 0.0302 | 73.9545 |
| 0.0003 | 1.0766 | 0.0063 | 21.1669 | 0.0123 | 38.5127 | 0.0183 | 53.1141 | 0.0243 | 64.9711 | 0.0303 | 74.0838 |
| 0.0004 | 1.4339 | 0.0064 | 21.4785 | 0.0124 | 38.7785 | 0.0184 | 53.3342 | 0.0244 | 65.1455 | 0.0304 | 74.2125 |
| 0.0005 | 1.7905 | 0.0065 | 21.7893 | 0.0125 | 39.0436 | 0.0185 | 53.5536 | 0.0245 | 65.3191 | 0.0305 | 74.3403 |
| 0.0006 | 2.1463 | 0.0066 | 22.0994 | 0.0126 | 39.3080 | 0.0186 | 53.7721 | 0.0246 | 65.4920 | 0.0306 | 74.4674 |
| 0.0007 | 2.5013 | 0.0067 | 22.4087 | 0.0127 | 39.5715 | 0.0187 | 53.9900 | 0.0247 | 65.6640 | 0.0307 | 74.5938 |
| 0.0008 | 2.8556 | 0.0068 | 22.7172 | 0.0128 | 39.8343 | 0.0188 | 54.2070 | 0.0248 | 65.8354 | 0.0308 | 74.7194 |
| 0.0009 | 3.2091 | 0.0069 | 23.0250 | 0.0129 | 40.0963 | 0.0189 | 54.4233 | 0.0249 | 66.0059 | 0.0309 | 74.8442 |
| 0.0010 | 3.5619 | 0.0070 | 23.3320 | 0.0130 | 40.3576 | 0.0190 | 54.6388 | 0.0250 | 66.1757 | 0.0310 | 74.9682 |
| 0.0011 | 3.9139 | 0.0071 | 23.6383 | 0.0131 | 40.6181 | 0.0191 | 54.8536 | 0.0251 | 66.3447 | 0.0311 | 75.0915 |
| 0.0012 | 4.2651 | 0.0072 | 23.9438 | 0.0132 | 40.8779 | 0.0192 | 55.0676 | 0.0252 | 66.5130 | 0.0312 | 75.2141 |
| 0.0013 | 4.6156 | 0.0073 | 24.2485 | 0.0133 | 41.1368 | 0.0193 | 55.2809 | 0.0253 | 66.6805 | 0.0313 | 75.3358 |
| 0.0014 | 4.9653 | 0.0074 | 24.5524 | 0.0134 | 41.3951 | 0.0194 | 55.4933 | 0.0254 | 66.8473 | 0.0314 | 75.4568 |
| 0.0015 | 5.3142 | 0.0075 | 24.8556 | 0.0135 | 41.6525 | 0.0195 | 55.7051 | 0.0255 | 67.0132 | 0.0315 | 75.5771 |
| 0.0016 | 5.6624 | 0.0076 | 25.1581 | 0.0136 | 41.9092 | 0.0196 | 55.9160 | 0.0256 | 67.1785 | 0.0316 | 75.6965 |
| 0.0017 | 6.0098 | 0.0077 | 25.4597 | 0.0137 | 42.1652 | 0.0197 | 56.1262 | 0.0257 | 67.3429 | 0.0317 | 75.8152 |
| 0.0018 | 6.3565 | 0.0078 | 25.7607 | 0.0138 | 42.4203 | 0.0198 | 56.3356 | 0.0258 | 67.5066 | 0.0318 | 75.9332 |
| 0.0019 | 6.7024 | 0.0079 | 26.0608 | 0.0139 | 42.6747 | 0.0199 | 56.5443 | 0.0259 | 67.6695 | 0.0319 | 76.0504 |
| 0.0020 | 7.0475 | 0.0080 | 26.3602 | 0.0140 | 42.9284 | 0.0200 | 56.7522 | 0.0260 | 67.8317 | 0.0320 | 76.1668 |
| 0.0021 | 7.3919 | 0.0081 | 26.6588 | 0.0141 | 43.1813 | 0.0201 | 56.9594 | 0.0261 | 67.9931 | 0.0321 | 76.2825 |
| 0.0022 | 7.7355 | 0.0082 | 26.9567 | 0.0142 | 43.4334 | 0.0202 | 57.1658 | 0.0262 | 68.1538 | 0.0322 | 76.3974 |
| 0.0023 | 8.0783 | 0.0083 | 27.2538 | 0.0143 | 43.6848 | 0.0203 | 57.3714 | 0.0263 | 68.3136 | 0.0323 | 76.5115 |
| 0.0024 | 8.4204 | 0.0084 | 27.5501 | 0.0144 | 43.9354 | 0.0204 | 57.5763 | 0.0264 | 68.4728 | 0.0324 | 76.6249 |
| 0.0025 | 8.7618 | 0.0085 | 27.8457 | 0.0145 | 44.1852 | 0.0205 | 57.7803 | 0.0265 | 68.6311 | 0.0325 | 76.7375 |
| 0.0026 | 9.1023 | 0.0086 | 28.1405 | 0.0146 | 44.4343 | 0.0206 | 57.9837 | 0.0266 | 68.7887 | 0.0326 | 76.8494 |
| 0.0027 | 9.4421 | 0.0087 | 28.4346 | 0.0147 | 44.6826 | 0.0207 | 58.1862 | 0.0267 | 68.9455 | 0.0327 | 76.9605 |
| 0.0028 | 9.7811 | 0.0088 | 28.7278 | 0.0148 | 44.9301 | 0.0208 | 58.3881 | 0.0268 | 69.1016 | 0.0328 | 77.0708 |
| 0.0029 | 10.1194 | 0.0089 | 29.0204 | 0.0149 | 45.1769 | 0.0209 | 58.5891 | 0.0269 | 69.2569 | 0.0329 | 77.1804 |
| 0.0030 | 10.4569 | 0.0090 | 29.3121 | 0.0150 | 45.4229 | 0.0210 | 58.7894 | 0.0270 | 69.4115 | 0.0330 | 77.2892 |
| 0.0031 | 10.7937 | 0.0091 | 29.6031 | 0.0151 | 45.6682 | 0.0211 | 58.9889 | 0.0271 | 69.5653 | 0.0331 | 77.3972 |
| 0.0032 | 11.1297 | 0.0092 | 29.8934 | 0.0152 | 45.9127 | 0.0212 | 59.1877 | 0.0272 | 69.7183 | 0.0332 | 77.5045 |
| 0.0033 | 11.4649 | 0.0093 | 30.1829 | 0.0153 | 46.1564 | 0.0213 | 59.3857 | 0.0273 | 69.8705 | 0.0333 | 77.6111 |
| 0.0034 | 11.7993 | 0.0094 | 30.4716 | 0.0154 | 46.3994 | 0.0214 | 59.5829 | 0.0274 | 70.0220 | 0.0334 | 77.7168 |
| 0.0035 | 12.1330 | 0.0095 | 30.7595 | 0.0155 | 46.6416 | 0.0215 | 59.7794 | 0.0275 | 70.1728 | 0.0335 | 77.8218 |
| 0.0036 | 12.4660 | 0.0096 | 31.0467 | 0.0156 | 46.8831 | 0.0216 | 59.9751 | 0.0276 | 70.3228 | 0.0336 | 77.9260 |
| 0.0037 | 12.7981 | 0.0097 | 31.3331 | 0.0157 | 47.1238 | 0.0217 | 60.1700 | 0.0277 | 70.4720 | 0.0337 | 78.0295 |
| 0.0038 | 13.1296 | 0.0098 | 31.6188 | 0.0158 | 47.3637 | 0.0218 | 60.3642 | 0.0278 | 70.6204 | 0.0338 | 78.1322 |
| 0.0039 | 13.4602 | 0.0099 | 31.9037 | 0.0159 | 47.6029 | 0.0219 | 60.5577 | 0.0279 | 70.7681 | 0.0339 | 78.2342 |
| 0.0040 | 13.7901 | 0.0100 | 32.1879 | 0.0160 | 47.8413 | 0.0220 | 60.7503 | 0.0280 | 70.9150 | 0.0340 | 78.3353 |
| 0.0041 | 14.1192 | 0.0101 | 32.4712 | 0.0161 | 48.0789 | 0.0221 | 60.9422 | 0.0281 | 71.0612 | 0.0341 | 78.4358 |
| 0.0042 | 14.4476 | 0.0102 | 32.7538 | 0.0162 | 48.3158 | 0.0222 | 61.1334 | 0.0282 | 71.2066 | 0.0342 | 78.5354 |
| 0.0043 | 14.7752 | 0.0103 | 33.0357 | 0.0163 | 48.5519 | 0.0223 | 61.3237 | 0.0283 | 71.3512 | 0.0343 | 78.6343 |
| 0.0044 | 15.1020 | 0.0104 | 33.3168 | 0.0164 | 48.7872 | 0.0224 | 61.5134 | 0.0284 | 71.4951 | 0.0344 | 78.7325 |
| 0.0045 | 15.4281 | 0.0105 | 33.5971 | 0.0165 | 49.0218 | 0.0225 | 61.7022 | 0.0285 | 71.6382 | 0.0345 | 78.8298 |
| 0.0046 | 15.7534 | 0.0106 | 33.8767 | 0.0166 | 49.2557 | 0.0226 | 61.8903 | 0.0286 | 71.7805 | 0.0346 | 78.9264 |
| 0.0047 | 16.0779 | 0.0107 | 34.1555 | 0.0167 | 49.4887 | 0.0227 | 62.0776 | 0.0287 | 71.9221 | 0.0347 | 79.0223 |
| 0.0048 | 16.4017 | 0.0108 | 34.4335 | 0.0168 | 49.7210 | 0.0228 | 62.2642 | 0.0288 | 72.0629 | 0.0348 | 79.1174 |
| 0.0049 | 16.7247 | 0.0109 | 34.7108 | 0.0169 | 49.9526 | 0.0229 | 62.4500 | 0.0289 | 72.2030 | 0.0349 | 79.2117 |
| 0.0050 | 17.0470 | 0.0110 | 34.9873 | 0.0170 | 50.1834 | 0.0230 | 62.6350 | 0.0290 | 72.3423 | 0.0350 | 79.3053 |
| 0.0051 | 17.3685 | 0.0111 | 35.2631 | 0.0171 | 50.4134 | 0.0231 | 62.8193 | 0.0291 | 72.4809 | 0.0351 | 79.3981 |
| 0.0052 | 17.6892 | 0.0112 | 35.5381 | 0.0172 | 50.6426 | 0.0232 | 63.0028 | 0.0292 | 72.6186 | 0.0352 | 79.4901 |
| 0.0053 | 18.0092 | 0.0113 | 35.8123 | 0.0173 | 50.8711 | 0.0233 | 63.1856 | 0.0293 | 72.7556 | 0.0353 | 79.5814 |
| 0.0054 | 18.3284 | 0.0114 | 36.0858 | 0.0174 | 51.0988 | 0.0234 | 63.3676 | 0.0294 | 72.8919 | 0.0354 | 79.6719 |
| 0.0055 | 18.6468 | 0.0115 | 36.3585 | 0.0175 | 51.3258 | 0.0235 | 63.5488 | 0.0295 | 73.0274 | 0.0355 | 79.7616 |
| 0.0056 | 18.9645 | 0.0116 | 36.6304 | 0.0176 | 51.5520 | 0.0236 | 63.7292 | 0.0296 | 73.1621 | 0.0356 | 79.8506 |
| 0.0057 | 19.2814 | 0.0117 | 36.9016 | 0.0177 | 51.7775 | 0.0237 | 63.9090 | 0.0297 | 73.2961 | 0.0357 | 79.9388 |
| 0.0058 | 19.5976 | 0.0118 | 37.1720 | 0.0178 | 52.0022 | 0.0238 | 64.0879 | 0.0298 | 73.4293 | 0.0358 | 80.0263 |
| 0.0059 | 19.9130 | 0.0119 | 37.4417 | 0.0179 | 52.2261 | 0.0239 | 64.2661 | 0.0299 | 73.5617 | 0.0359 | 80.1130 |

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FC= 250.0000 FY= 3600.0000

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| ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.0060 | 20.5020 | 0.0120 | 38.8083 | 0.0180 | 54.9192 | 0.0240 | 68.8345 | 0.0300 | 80.5543 |
| 0.0001 | 0.3597 | 0.0061 | 20.8251 | 0.0121 | 39.0948 | 0.0181 | 55.1691 | 0.0241 | 69.0478 | 0.0301 | 80.7311 |
| 0.0002 | 0.7188 | 0.0062 | 21.1476 | 0.0122 | 39.3808 | 0.0182 | 55.4184 | 0.0242 | 69.2605 | 0.0302 | 80.9072 |
| 0.0003 | 1.0773 | 0.0063 | 21.4695 | 0.0123 | 39.6660 | 0.0183 | 55.6671 | 0.0243 | 69.4726 | 0.0303 | 81.0827 |
| 0.0004 | 1.4351 | 0.0064 | 21.7908 | 0.0124 | 39.9507 | 0.0184 | 55.9151 | 0.0244 | 69.6841 | 0.0304 | 81.2576 |
| 0.0005 | 1.7924 | 0.0065 | 22.1114 | 0.0125 | 40.2348 | 0.0185 | 56.1626 | 0.0245 | 69.8950 | 0.0305 | 81.4319 |
| 0.0006 | 2.1490 | 0.0066 | 22.4315 | 0.0126 | 40.5182 | 0.0186 | 56.4095 | 0.0246 | 70.1053 | 0.0306 | 81.6056 |
| 0.0007 | 2.5051 | 0.0067 | 22.7509 | 0.0127 | 40.8011 | 0.0187 | 56.6557 | 0.0247 | 70.3149 | 0.0307 | 81.7786 |
| 0.0008 | 2.8605 | 0.0068 | 23.0697 | 0.0128 | 41.0833 | 0.0188 | 56.9014 | 0.0248 | 70.5240 | 0.0308 | 81.9511 |
| 0.0009 | 3.2153 | 0.0069 | 23.3880 | 0.0129 | 41.3649 | 0.0189 | 57.1464 | 0.0249 | 70.7324 | 0.0309 | 82.1229 |
| 0.0010 | 3.5695 | 0.0070 | 23.7056 | 0.0130 | 41.6460 | 0.0190 | 57.3908 | 0.0250 | 70.9402 | 0.0310 | 82.2942 |
| 0.0011 | 3.9231 | 0.0071 | 24.0226 | 0.0131 | 41.9263 | 0.0191 | 57.6347 | 0.0251 | 71.1475 | 0.0311 | 82.4648 |
| 0.0012 | 4.2761 | 0.0072 | 24.3389 | 0.0132 | 42.2062 | 0.0192 | 57.8779 | 0.0252 | 71.3541 | 0.0312 | 82.6348 |
| 0.0013 | 4.6285 | 0.0073 | 24.6547 | 0.0133 | 42.4853 | 0.0193 | 58.1205 | 0.0253 | 71.5601 | 0.0313 | 82.8042 |
| 0.0014 | 4.9802 | 0.0074 | 24.9699 | 0.0134 | 42.7639 | 0.0194 | 58.3624 | 0.0254 | 71.7655 | 0.0314 | 82.9731 |
| 0.0015 | 5.3314 | 0.0075 | 25.2844 | 0.0135 | 43.0419 | 0.0195 | 58.6038 | 0.0255 | 71.9703 | 0.0315 | 83.1412 |
| 0.0016 | 5.6819 | 0.0076 | 25.5984 | 0.0136 | 43.3192 | 0.0196 | 58.8446 | 0.0256 | 72.1746 | 0.0316 | 83.3088 |
| 0.0017 | 6.0319 | 0.0077 | 25.9117 | 0.0137 | 43.5960 | 0.0197 | 59.0847 | 0.0257 | 72.3780 | 0.0317 | 83.4758 |
| 0.0018 | 6.3812 | 0.0078 | 26.2245 | 0.0138 | 43.8721 | 0.0198 | 59.3243 | 0.0258 | 72.5810 | 0.0318 | 83.6422 |
| 0.0019 | 6.7299 | 0.0079 | 26.5366 | 0.0139 | 44.1476 | 0.0199 | 59.5632 | 0.0259 | 72.7833 | 0.0319 | 83.8079 |
| 0.0020 | 7.0780 | 0.0080 | 26.8481 | 0.0140 | 44.4226 | 0.0200 | 59.8015 | 0.0260 | 72.9850 | 0.0320 | 83.9731 |
| 0.0021 | 7.4255 | 0.0081 | 27.1590 | 0.0141 | 44.6969 | 0.0201 | 60.0393 | 0.0261 | 73.1862 | 0.0321 | 84.1376 |
| 0.0022 | 7.7724 | 0.0082 | 27.4693 | 0.0142 | 44.9706 | 0.0202 | 60.2764 | 0.0262 | 73.3867 | 0.0322 | 84.3015 |
| 0.0023 | 8.1187 | 0.0083 | 27.7790 | 0.0143 | 45.2437 | 0.0203 | 60.5129 | 0.0263 | 73.5866 | 0.0323 | 84.4648 |
| 0.0024 | 8.4643 | 0.0084 | 28.0880 | 0.0144 | 45.5161 | 0.0204 | 60.7487 | 0.0264 | 73.7859 | 0.0324 | 84.6275 |
| 0.0025 | 8.8094 | 0.0085 | 28.3965 | 0.0145 | 45.7880 | 0.0205 | 60.9840 | 0.0265 | 73.9846 | 0.0325 | 84.7896 |
| 0.0026 | 9.1538 | 0.0086 | 28.7043 | 0.0146 | 46.0592 | 0.0206 | 61.2187 | 0.0266 | 74.1826 | 0.0326 | 84.9511 |
| 0.0027 | 9.4977 | 0.0087 | 29.0116 | 0.0147 | 46.3299 | 0.0207 | 61.4528 | 0.0267 | 74.3801 | 0.0327 | 85.1120 |
| 0.0028 | 9.8409 | 0.0088 | 29.3182 | 0.0148 | 46.5999 | 0.0208 | 61.6862 | 0.0268 | 74.5770 | 0.0328 | 85.2727 |
| 0.0029 | 10.1835 | 0.0089 | 29.6242 | 0.0149 | 46.8694 | 0.0209 | 61.9190 | 0.0269 | 74.7732 | 0.0329 | 85.4319 |
| 0.0030 | 10.5255 | 0.0090 | 29.9296 | 0.0150 | 47.1382 | 0.0210 | 62.1513 | 0.0270 | 74.9688 | 0.0330 | 85.5909 |
| 0.0031 | 10.8669 | 0.0091 | 30.2344 | 0.0151 | 47.4064 | 0.0211 | 62.3829 | 0.0271 | 75.1639 | 0.0331 | 85.7494 |
| 0.0032 | 11.2077 | 0.0092 | 30.5386 | 0.0152 | 47.6740 | 0.0212 | 62.6139 | 0.0272 | 75.3583 | 0.0332 | 85.9072 |
| 0.0033 | 11.5479 | 0.0093 | 30.8422 | 0.0153 | 47.9410 | 0.0213 | 62.8443 | 0.0273 | 75.5521 | 0.0333 | 86.0644 |
| 0.0034 | 11.8875 | 0.0094 | 31.1452 | 0.0154 | 48.2074 | 0.0214 | 63.0741 | 0.0274 | 75.7453 | 0.0334 | 86.2210 |
| 0.0035 | 12.2264 | 0.0095 | 31.4475 | 0.0155 | 48.4731 | 0.0215 | 63.3032 | 0.0275 | 75.9379 | 0.0335 | 86.3770 |
| 0.0036 | 12.5648 | 0.0096 | 31.7493 | 0.0156 | 48.7383 | 0.0216 | 63.5318 | 0.0276 | 76.1299 | 0.0336 | 86.5324 |
| 0.0037 | 12.9025 | 0.0097 | 32.0504 | 0.0157 | 49.0029 | 0.0217 | 63.7598 | 0.0277 | 76.3212 | 0.0337 | 86.6872 |
| 0.0038 | 13.2396 | 0.0098 | 32.3510 | 0.0158 | 49.2668 | 0.0218 | 63.9871 | 0.0278 | 76.5120 | 0.0338 | 86.8413 |
| 0.0039 | 13.5762 | 0.0099 | 32.6509 | 0.0159 | 49.5301 | 0.0219 | 64.2139 | 0.0279 | 76.7021 | 0.0339 | 86.9949 |
| 0.0040 | 13.9121 | 0.0100 | 32.9502 | 0.0160 | 49.7928 | 0.0220 | 64.4400 | 0.0280 | 76.8917 | 0.0340 | 87.1478 |
| 0.0041 | 14.2474 | 0.0101 | 33.2489 | 0.0161 | 50.0549 | 0.0221 | 64.6655 | 0.0281 | 77.0806 | 0.0341 | 87.3002 |
| 0.0042 | 14.5820 | 0.0102 | 33.5470 | 0.0162 | 50.3164 | 0.0222 | 64.8904 | 0.0282 | 77.2689 | 0.0342 | 87.4519 |
| 0.0043 | 14.9161 | 0.0103 | 33.8445 | 0.0163 | 50.5773 | 0.0223 | 65.1147 | 0.0283 | 77.4566 | 0.0343 | 87.6030 |
| 0.0044 | 15.2496 | 0.0104 | 34.1413 | 0.0164 | 50.8376 | 0.0224 | 65.3384 | 0.0284 | 77.6437 | 0.0344 | 87.7535 |
| 0.0045 | 15.5824 | 0.0105 | 34.4376 | 0.0165 | 51.0973 | 0.0225 | 65.5615 | 0.0285 | 77.8302 | 0.0345 | 87.9034 |
| 0.0046 | 15.9147 | 0.0106 | 34.7332 | 0.0166 | 51.3564 | 0.0226 | 65.7840 | 0.0286 | 78.0161 | 0.0346 | 88.0527 |
| 0.0047 | 16.2463 | 0.0107 | 35.0283 | 0.0167 | 51.6148 | 0.0227 | 66.0058 | 0.0287 | 78.2013 | 0.0347 | 88.2014 |
| 0.0048 | 16.5773 | 0.0108 | 35.3227 | 0.0168 | 51.8726 | 0.0228 | 66.2271 | 0.0288 | 78.3860 | 0.0348 | 88.3494 |
| 0.0049 | 16.9077 | 0.0109 | 35.6165 | 0.0169 | 52.1299 | 0.0229 | 66.4477 | 0.0289 | 78.5700 | 0.0349 | 88.4969 |
| 0.0050 | 17.2375 | 0.0110 | 35.9098 | 0.0170 | 52.3865 | 0.0230 | 66.6677 | 0.0290 | 78.7535 | 0.0350 | 88.6438 |
| 0.0051 | 17.5667 | 0.0111 | 36.2024 | 0.0171 | 52.6425 | 0.0231 | 66.8871 | 0.0291 | 78.9363 | 0.0351 | 88.7900 |
| 0.0052 | 17.8953 | 0.0112 | 36.4944 | 0.0172 | 52.8979 | 0.0232 | 67.1060 | 0.0292 | 79.1185 | 0.0352 | 88.9356 |
| 0.0053 | 18.2233 | 0.0113 | 36.7857 | 0.0173 | 53.1527 | 0.0233 | 67.3242 | 0.0293 | 79.3001 | 0.0353 | 89.0806 |
| 0.0054 | 18.5507 | 0.0114 | 37.0765 | 0.0174 | 53.4069 | 0.0234 | 67.5417 | 0.0294 | 79.4811 | 0.0354 | 89.2250 |
| 0.0055 | 18.8774 | 0.0115 | 37.3667 | 0.0175 | 53.6604 | 0.0235 | 67.7587 | 0.0295 | 79.6615 | 0.0355 | 89.3688 |
| 0.0056 | 19.2036 | 0.0116 | 37.6562 | 0.0176 | 53.9134 | 0.0236 | 67.9751 | 0.0296 | 79.8413 | 0.0356 | 89.5120 |
| 0.0057 | 19.5291 | 0.0117 | 37.9452 | 0.0177 | 54.1658 | 0.0237 | 68.1909 | 0.0297 | 80.0205 | 0.0357 | 89.6546 |
| 0.0058 | 19.8540 | 0.0118 | 38.2335 | 0.0178 | 54.4175 | 0.0238 | 68.4060 | 0.0298 | 80.1990 | 0.0358 | 89.7966 |
| 0.0059 | 20.1783 | 0.0119 | 38.5212 | 0.0179 | 54.6686 | 0.0239 | 68.6206 | 0.0299 | 80.3770 | 0.0359 | 89.9379 |

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FC1= 160.0000 FY= 2300.0000

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| ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.0060 | 13.0998 | 0.0120 | 24.7991 | 0.0180 | 35.0982 | 0.0240 | 43.9971 | 0.0300 | 51.4957 |
| 0.0001 | 0.2298 | 0.0061 | 13.3062 | 0.0121 | 24.9822 | 0.0181 | 35.2580 | 0.0241 | 44.1335 | 0.0301 | 51.6088 |
| 0.0002 | 0.4592 | 0.0062 | 13.5123 | 0.0122 | 25.1649 | 0.0182 | 35.4174 | 0.0242 | 44.2696 | 0.0302 | 51.7215 |
| 0.0003 | 0.6882 | 0.0063 | 13.7180 | 0.0123 | 25.3473 | 0.0183 | 35.5764 | 0.0243 | 44.4052 | 0.0303 | 51.8339 |
| 0.0004 | 0.9169 | 0.0064 | 13.9233 | 0.0124 | 25.5293 | 0.0184 | 35.7350 | 0.0244 | 44.5405 | 0.0304 | 51.9458 |
| 0.0005 | 1.1451 | 0.0065 | 14.1282 | 0.0125 | 25.7108 | 0.0185 | 35.8932 | 0.0245 | 44.6754 | 0.0305 | 52.0574 |
| 0.0006 | 1.3730 | 0.0066 | 14.3327 | 0.0126 | 25.8920 | 0.0186 | 36.0511 | 0.0246 | 44.8099 | 0.0306 | 52.1685 |
| 0.0007 | 1.6005 | 0.0067 | 14.5368 | 0.0127 | 26.0728 | 0.0187 | 36.2085 | 0.0247 | 44.9440 | 0.0307 | 52.2793 |
| 0.0008 | 1.8275 | 0.0068 | 14.7406 | 0.0128 | 26.2532 | 0.0188 | 36.3656 | 0.0248 | 45.0778 | 0.0308 | 52.3897 |
| 0.0009 | 2.0542 | 0.0069 | 14.9439 | 0.0129 | 26.4332 | 0.0189 | 36.5223 | 0.0249 | 45.2111 | 0.0309 | 52.4997 |
| 0.0010 | 2.2805 | 0.0070 | 15.1469 | 0.0130 | 26.6128 | 0.0190 | 36.6786 | 0.0250 | 45.3441 | 0.0310 | 52.6093 |
| 0.0011 | 2.5065 | 0.0071 | 15.3495 | 0.0131 | 26.7921 | 0.0191 | 36.8345 | 0.0251 | 45.4766 | 0.0311 | 52.7186 |
| 0.0012 | 2.7320 | 0.0072 | 15.5516 | 0.0132 | 26.9709 | 0.0192 | 36.9900 | 0.0252 | 45.6088 | 0.0312 | 52.8274 |
| 0.0013 | 2.9571 | 0.0073 | 15.7534 | 0.0133 | 27.1494 | 0.0193 | 37.1451 | 0.0253 | 45.7406 | 0.0313 | 52.9358 |
| 0.0014 | 3.1819 | 0.0074 | 15.9548 | 0.0134 | 27.3275 | 0.0194 | 37.2999 | 0.0254 | 45.8720 | 0.0314 | 53.0439 |
| 0.0015 | 3.4062 | 0.0075 | 16.1559 | 0.0135 | 27.5051 | 0.0195 | 37.4542 | 0.0255 | 46.0030 | 0.0315 | 53.1516 |
| 0.0016 | 3.6302 | 0.0076 | 16.3565 | 0.0136 | 27.6824 | 0.0196 | 37.6081 | 0.0256 | 46.1336 | 0.0316 | 53.2588 |
| 0.0017 | 3.8538 | 0.0077 | 16.5567 | 0.0137 | 27.8593 | 0.0197 | 37.7617 | 0.0257 | 46.2638 | 0.0317 | 53.3657 |
| 0.0018 | 4.0770 | 0.0078 | 16.7566 | 0.0138 | 28.0358 | 0.0198 | 37.9149 | 0.0258 | 46.3937 | 0.0318 | 53.4722 |
| 0.0019 | 4.2998 | 0.0079 | 16.9560 | 0.0139 | 28.2120 | 0.0199 | 38.0677 | 0.0259 | 46.5231 | 0.0319 | 53.5783 |
| 0.0020 | 4.5222 | 0.0080 | 17.1551 | 0.0140 | 28.3877 | 0.0200 | 38.2201 | 0.0260 | 46.6522 | 0.0320 | 53.6841 |
| 0.0021 | 4.7442 | 0.0081 | 17.3538 | 0.0141 | 28.5630 | 0.0201 | 38.3721 | 0.0261 | 46.7809 | 0.0321 | 53.7894 |
| 0.0022 | 4.9659 | 0.0082 | 17.5521 | 0.0142 | 28.7380 | 0.0202 | 38.5237 | 0.0262 | 46.9091 | 0.0322 | 53.8943 |
| 0.0023 | 5.1871 | 0.0083 | 17.7500 | 0.0143 | 28.9126 | 0.0203 | 38.6749 | 0.0263 | 47.0370 | 0.0323 | 53.9989 |
| 0.0024 | 5.4080 | 0.0084 | 17.9475 | 0.0144 | 29.0868 | 0.0204 | 38.8258 | 0.0264 | 47.1646 | 0.0324 | 54.1031 |
| 0.0025 | 5.6284 | 0.0085 | 18.1446 | 0.0145 | 29.2605 | 0.0205 | 38.9763 | 0.0265 | 47.2917 | 0.0325 | 54.2069 |
| 0.0026 | 5.8485 | 0.0086 | 18.3414 | 0.0146 | 29.4340 | 0.0206 | 39.1263 | 0.0266 | 47.4184 | 0.0326 | 54.3102 |
| 0.0027 | 6.0682 | 0.0087 | 18.5377 | 0.0147 | 29.6070 | 0.0207 | 39.2760 | 0.0267 | 47.5447 | 0.0327 | 54.4133 |
| 0.0028 | 6.2875 | 0.0088 | 18.7337 | 0.0148 | 29.7796 | 0.0208 | 39.4252 | 0.0268 | 47.6707 | 0.0328 | 54.5159 |
| 0.0029 | 6.5064 | 0.0089 | 18.9293 | 0.0149 | 29.9518 | 0.0209 | 39.5741 | 0.0269 | 47.7962 | 0.0329 | 54.6181 |
| 0.0030 | 6.7250 | 0.0090 | 19.1244 | 0.0150 | 30.1237 | 0.0210 | 39.7227 | 0.0270 | 47.9214 | 0.0330 | 54.7199 |
| 0.0031 | 6.9431 | 0.0091 | 19.3192 | 0.0151 | 30.2951 | 0.0211 | 39.8708 | 0.0271 | 48.0462 | 0.0331 | 54.8214 |
| 0.0032 | 7.1608 | 0.0092 | 19.5136 | 0.0152 | 30.4662 | 0.0212 | 40.0185 | 0.0272 | 48.1706 | 0.0332 | 54.9224 |
| 0.0033 | 7.3782 | 0.0093 | 19.7077 | 0.0153 | 30.6369 | 0.0213 | 40.1658 | 0.0273 | 48.2946 | 0.0333 | 55.0231 |
| 0.0034 | 7.5952 | 0.0094 | 19.9013 | 0.0154 | 30.8072 | 0.0214 | 40.3128 | 0.0274 | 48.4182 | 0.0334 | 55.1234 |
| 0.0035 | 7.8117 | 0.0095 | 20.0945 | 0.0155 | 30.9771 | 0.0215 | 40.4594 | 0.0275 | 48.5414 | 0.0335 | 55.2233 |
| 0.0036 | 8.0279 | 0.0096 | 20.2874 | 0.0156 | 31.1466 | 0.0216 | 40.6055 | 0.0276 | 48.6643 | 0.0336 | 55.3228 |
| 0.0037 | 8.2437 | 0.0097 | 20.4798 | 0.0157 | 31.3157 | 0.0217 | 40.7513 | 0.0277 | 48.7867 | 0.0337 | 55.4219 |
| 0.0038 | 8.4591 | 0.0098 | 20.6719 | 0.0158 | 31.4844 | 0.0218 | 40.8967 | 0.0278 | 48.9088 | 0.0338 | 55.5206 |
| 0.0039 | 8.6742 | 0.0099 | 20.8636 | 0.0159 | 31.6528 | 0.0219 | 41.0417 | 0.0279 | 49.0304 | 0.0339 | 55.6189 |
| 0.0040 | 8.8888 | 0.0100 | 21.0549 | 0.0160 | 31.8207 | 0.0220 | 41.1864 | 0.0280 | 49.1517 | 0.0340 | 55.7169 |
| 0.0041 | 9.1031 | 0.0101 | 21.2458 | 0.0161 | 31.9883 | 0.0221 | 41.3306 | 0.0281 | 49.2726 | 0.0341 | 55.8144 |
| 0.0042 | 9.3169 | 0.0102 | 21.4363 | 0.0162 | 32.1555 | 0.0222 | 41.4744 | 0.0282 | 49.3931 | 0.0342 | 55.9116 |
| 0.0043 | 9.5304 | 0.0103 | 21.6264 | 0.0163 | 32.3223 | 0.0223 | 41.6179 | 0.0283 | 49.5132 | 0.0343 | 56.0084 |
| 0.0044 | 9.7434 | 0.0104 | 21.8162 | 0.0164 | 32.4887 | 0.0224 | 41.7609 | 0.0284 | 49.6330 | 0.0344 | 56.1048 |
| 0.0045 | 9.9561 | 0.0105 | 22.0055 | 0.0165 | 32.6547 | 0.0225 | 41.9036 | 0.0285 | 49.7523 | 0.0345 | 56.2008 |
| 0.0046 | 10.1684 | 0.0106 | 22.1945 | 0.0166 | 32.8203 | 0.0226 | 42.0459 | 0.0286 | 49.8712 | 0.0346 | 56.2964 |
| 0.0047 | 10.3803 | 0.0107 | 22.3831 | 0.0167 | 32.9855 | 0.0227 | 42.1878 | 0.0287 | 49.9898 | 0.0347 | 56.3916 |
| 0.0048 | 10.5919 | 0.0108 | 22.5712 | 0.0168 | 33.1504 | 0.0228 | 42.3293 | 0.0288 | 50.1080 | 0.0348 | 56.4864 |
| 0.0049 | 10.8030 | 0.0109 | 22.7590 | 0.0169 | 33.3148 | 0.0229 | 42.4704 | 0.0289 | 50.2258 | 0.0349 | 56.5809 |
| 0.0050 | 11.0137 | 0.0110 | 22.9464 | 0.0170 | 33.4789 | 0.0230 | 42.6112 | 0.0290 | 50.3432 | 0.0350 | 56.6749 |
| 0.0051 | 11.2241 | 0.0111 | 23.1335 | 0.0171 | 33.6426 | 0.0231 | 42.7515 | 0.0291 | 50.4602 | 0.0351 | 56.7686 |
| 0.0052 | 11.4340 | 0.0112 | 23.3201 | 0.0172 | 33.8059 | 0.0232 | 42.8914 | 0.0292 | 50.5768 | 0.0352 | 56.8619 |
| 0.0053 | 11.6436 | 0.0113 | 23.5063 | 0.0173 | 33.9688 | 0.0233 | 43.0310 | 0.0293 | 50.6930 | 0.0353 | 56.9548 |
| 0.0054 | 11.8528 | 0.0114 | 23.6922 | 0.0174 | 34.1313 | 0.0234 | 43.1702 | 0.0294 | 50.8088 | 0.0354 | 57.0473 |
| 0.0055 | 12.0616 | 0.0115 | 23.8776 | 0.0175 | 34.2934 | 0.0235 | 43.3090 | 0.0295 | 50.9243 | 0.0355 | 57.1394 |
| 0.0056 | 12.2700 | 0.0116 | 24.0627 | 0.0176 | 34.4551 | 0.0236 | 43.4474 | 0.0296 | 51.0394 | 0.0356 | 57.2311 |
| 0.0057 | 12.4780 | 0.0117 | 24.2474 | 0.0177 | 34.6165 | 0.0237 | 43.5854 | 0.0297 | 51.1540 | 0.0357 | 57.3224 |
| 0.0058 | 12.6857 | 0.0118 | 24.4317 | 0.0178 | 34.7775 | 0.0238 | 43.7230 | 0.0298 | 51.2683 | 0.0358 | 57.4134 |
| 0.0059 | 12.8929 | 0.0119 | 24.6156 | 0.0179 | 34.9380 | 0.0239 | 43.8602 | 0.0299 | 51.3822 | 0.0359 | 57.5039 |

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FC1= 180.0000 FY= 2300.0000

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| ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN | ROW | RN |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.0060 | 13.1775 | 0.0120 | 25.1103 | 0.0180 | 35.7983 | 0.0240 | 45.2417 | 0.0300 | 53.4405 |
| 0.0001 | 0.2298 | b 0.0061 | 13.3866 | 0.0121 | 25.2986 | 0.0181 | 35.9659 | 0.0241 | 45.3886 | 0.0301 | 53.5666 |
| 0.0002 | 0.4593 | 0.0062 | 13.5954 | 0.0122 | 25.4866 | 0.0182 | 36.1331 | 0.0242 | 45.5351 | 0.0302 | 53.6923 |
| 0.0003 | 0.6884 | 0.0063 | 13.8037 | 0.0123 | 25.6742 | 0.0183 | 36.3000 | 0.0243 | 45.6812 | 0.0303 | 53.8177 |
| 0.0004 | 0.9172 | 0.0064 | 14.0118 | 0.0124 | 25.8615 | 0.0184 | 36.4666 | 0.0244 | 45.8270 | 0.0304 | 53.9428 |
| 0.0005 | 1.1457 | 0.0065 | 14.2195 | 0.0125 | 26.0485 | 0.0185 | 36.6328 | 0.0245 | 45.9725 | 0.0305 | 54.0675 |
| 0.0006 | 1.3738 | 0.0066 | 14.4268 | 0.0126 | 26.2351 | 0.0186 | 36.7987 | 0.0246 | 46.1176 | 0.0306 | 54.1919 |
| 0.0007 | 1.6015 | 0.0067 | 14.6338 | 0.0127 | 26.4213 | 0.0187 | 36.9642 | 0.0247 | 46.2624 | 0.0307 | 54.3159 |
| 0.0008 | 1.8289 | 0.0068 | 14.8405 | 0.0128 | 26.6072 | 0.0188 | 37.1293 | 0.0248 | 46.4068 | 0.0308 | 54.4396 |
| 0.0009 | 2.0560 | 0.0069 | 15.0468 | 0.0129 | 26.7928 | 0.0189 | 37.2942 | 0.0249 | 46.5509 | 0.0309 | 54.5629 |
| 0.0010 | 2.2827 | 0.0070 | 15.2528 | 0.0130 | 26.9780 | 0.0190 | 37.4587 | 0.0250 | 46.6946 | 0.0310 | 54.6859 |
| 0.0011 | 2.5091 | 0.0071 | 15.4584 | 0.0131 | 27.1629 | 0.0191 | 37.6228 | 0.0251 | 46.8380 | 0.0311 | 54.8086 |
| 0.0012 | 2.7351 | 0.0072 | 15.6637 | 0.0132 | 27.3474 | 0.0192 | 37.7866 | 0.0252 | 46.9810 | 0.0312 | 54.9309 |
| 0.0013 | 2.9608 | 0.0073 | 15.8686 | 0.0133 | 27.5316 | 0.0193 | 37.9500 | 0.0253 | 47.1237 | 0.0313 | 55.0528 |
| 0.0014 | 3.1861 | 0.0074 | 16.0732 | 0.0134 | 27.7155 | 0.0194 | 38.1131 | 0.0254 | 47.2661 | 0.0314 | 55.1744 |
| 0.0015 | 3.4111 | 0.0075 | 16.2774 | 0.0135 | 27.8990 | 0.0195 | 38.2759 | 0.0255 | 47.4081 | 0.0315 | 55.2957 |
| 0.0016 | 3.6357 | 0.0076 | 16.4813 | 0.0136 | 28.0821 | 0.0196 | 38.4383 | 0.0256 | 47.5498 | 0.0316 | 55.4166 |
| 0.0017 | 3.8600 | 0.0077 | 16.6848 | 0.0137 | 28.2649 | 0.0197 | 38.6003 | 0.0257 | 47.6911 | 0.0317 | 55.5372 |
| 0.0018 | 4.0840 | 0.0078 | 16.8880 | 0.0138 | 28.4474 | 0.0198 | 38.7620 | 0.0258 | 47.8320 | 0.0318 | 55.6574 |
| 0.0019 | 4.3076 | 0.0079 | 17.0909 | 0.0139 | 28.6295 | 0.0199 | 38.9234 | 0.0259 | 47.9727 | 0.0319 | 55.7773 |
| 0.0020 | 4.5308 | 0.0080 | 17.2934 | 0.0140 | 28.8112 | 0.0200 | 39.0844 | 0.0260 | 48.1129 | 0.0320 | 55.8968 |
| 0.0021 | 4.7538 | 0.0081 | 17.4956 | 0.0141 | 28.9927 | 0.0201 | 39.2451 | 0.0261 | 48.2529 | 0.0321 | 56.0160 |
| 0.0022 | 4.9763 | 0.0082 | 17.6974 | 0.0142 | 29.1737 | 0.0202 | 39.4054 | 0.0262 | 48.3925 | 0.0322 | 56.1348 |
| 0.0023 | 5.1985 | 0.0083 | 17.8989 | 0.0143 | 29.3545 | 0.0203 | 39.5654 | 0.0263 | 48.5317 | 0.0323 | 56.2533 |
| 0.0024 | 5.4204 | 0.0084 | 18.1000 | 0.0144 | 29.5348 | 0.0204 | 39.7250 | 0.0264 | 48.6706 | 0.0324 | 56.3715 |
| s 0.0025 | 5.6419 | 0.0085 | 18.3008 | 0.0145 | 29.7149 | 0.0205 | 39.8843 | 0.0265 | 48.8091 | 0.0325 | 56.4893 |
| 0.0026 | 5.8631 | 0.0086 | 18.5012 | 0.0146 | 29.8946 | 0.0206 | 40.0433 | 0.0266 | 48.9473 | 0.0326 | 56.6067 |
| 0.0027 | 6.0840 | 0.0087 | 18.7013 | 0.0147 | 30.0739 | 0.0207 | 40.2019 | 0.0267 | 49.0852 | 0.0327 | 56.7238 |
| 0.0028 | 6.3045 | 0.0088 | 18.9010 | 0.0148 | 30.2529 | 0.0208 | 40.3601 | 0.0268 | 49.2227 | 0.0328 | 56.8406 |
| 0.0029 | 6.5246 | 0.0089 | 19.1004 | 0.0149 | 30.4316 | 0.0209 | 40.5180 | 0.0269 | 49.3599 | 0.0329 | 56.9570 |
| 0.0030 | 6.7444 | 0.0090 | 19.2995 | 0.0150 | 30.6099 | 0.0210 | 40.6756 | 0.0270 | 49.4967 | 0.0330 | 57.0731 |
| 0.0031 | 6.9639 | 0.0091 | 19.4982 | 0.0151 | 30.7878 | 0.0211 | 40.8328 | 0.0271 | 49.6332 | 0.0331 | 57.1888 |
| 0.0032 | 7.1830 | 0.0092 | 19.6965 | 0.0152 | 30.9655 | 0.0212 | 40.9897 | 0.0272 | 49.7693 | 0.0332 | 57.3042 |
| 0.0033 | 7.4017 | 0.0093 | 19.8945 | 0.0153 | 31.1427 | 0.0213 | 41.1462 | 0.0273 | 49.9051 | 0.0333 | 57.4193 |
| 0.0034 | 7.6201 | 0.0094 | 20.0922 | 0.0154 | 31.3196 | 0.0214 | 41.3024 | 0.0274 | 50.0405 | 0.0334 | 57.5340 |
| 0.0035 | 7.8382 | 0.0095 | 20.2896 | 0.0155 | 31.4962 | 0.0215 | 41.4582 | 0.0275 | 50.1756 | 0.0335 | 57.6483 |
| 0.0036 | 8.0559 | 0.0096 | 20.4865 | 0.0156 | 31.6725 | 0.0216 | 41.6137 | 0.0276 | 50.3103 | 0.0336 | 57.7623 |
| 0.0037 | 8.2733 | 0.0097 | 20.6832 | 0.0157 | 31.8483 | 0.0217 | 41.7689 | 0.0277 | 50.4447 | 0.0337 | 57.8759 |
| 0.0038 | 8.4904 | 0.0098 | 20.8794 | 0.0158 | 32.0239 | 0.0218 | 41.9237 | 0.0278 | 50.5788 | 0.0338 | 57.9893 |
| 0.0039 | 8.7070 | 0.0099 | 21.0754 | 0.0159 | 32.1991 | 0.0219 | 42.0781 | 0.0279 | 50.7125 | 0.0339 | 58.1022 |
| 0.0040 | 8.9234 | 0.0100 | 21.2710 | 0.0160 | 32.3739 | 0.0220 | 42.2322 | 0.0280 | 50.8459 | 0.0340 | 58.2148 |
| 0.0041 | 9.1394 | 0.0101 | 21.4662 | 0.0161 | 32.5484 | 0.0221 | 42.3860 | 0.0281 | 50.9789 | 0.0341 | 58.3271 |
| 0.0042 | 9.3550 | 0.0102 | 21.6611 | 0.0162 | 32.7226 | 0.0222 | 42.5394 | 0.0282 | 51.1115 | 0.0342 | 58.4390 |
| 0.0043 | 9.5703 | 0.0103 | 21.8557 | 0.0163 | 32.8964 | 0.0223 | 42.6925 | 0.0283 | 51.2439 | 0.0343 | 58.5506 |
| 0.0044 | 9.7853 | 0.0104 | 22.0499 | 0.0164 | 33.0699 | 0.0224 | 42.8452 | 0.0284 | 51.3758 | 0.0344 | 58.6618 |
| 0.0045 | 9.9999 | 0.0105 | 22.2438 | 0.0165 | 33.2430 | 0.0225 | 42.9976 | 0.0285 | 51.5075 | 0.0345 | 58.7727 |
| 0.0046 | 10.2142 | 0.0106 | 22.4373 | 0.0166 | 33.4158 | 0.0226 | 43.1496 | 0.0286 | 51.6388 | 0.0346 | 58.8833 |
| 0.0047 | 10.4281 | 0.0107 | 22.6305 | 0.0167 | 33.5882 | 0.0227 | 43.3013 | 0.0287 | 51.7697 | 0.0347 | 58.9935 |
| 0.0048 | 10.6416 | 0.0108 | 22.8233 | 0.0168 | 33.7603 | 0.0228 | 43.4526 | 0.0288 | 51.9003 | 0.0348 | 59.1033 |
| 0.0049 | 10.8549 | 0.0109 | 23.0158 | 0.0169 | 33.9320 | 0.0229 | 43.6036 | 0.0289 | 52.0305 | 0.0349 | 59.2128 |
| 0.0050 | 11.0678 | 0.0110 | 23.2079 | 0.0170 | 34.1034 | 0.0230 | 43.7543 | 0.0290 | 52.1605 | 0.0350 | 59.3220 |
| 0.0051 | 11.2803 | 0.0111 | 23.3997 | 0.0171 | 34.2745 | 0.0231 | 43.9046 | 0.0291 | 52.2900 | 0.0351 | 59.4308 |
| 0.0052 | 11.4925 | 0.0112 | 23.5911 | 0.0172 | 34.4452 | 0.0232 | 44.0545 | 0.0292 | 52.4192 | 0.0352 | 59.5393 |
| 0.0053 | 11.7043 | 0.0113 | 23.7822 | 0.0173 | 34.6155 | 0.0233 | 44.2041 | 0.0293 | 52.5481 | 0.0353 | 59.6474 |
| 0.0054 | 11.9158 | 0.0114 | 23.9730 | 0.0174 | 34.7855 | 0.0234 | 44.3534 | 0.0294 | 52.6766 | 0.0354 | 59.7552 |
| 0.0055 | 12.1270 | 0.0115 | 24.1634 | 0.0175 | 34.9552 | 0.0235 | 44.5023 | 0.0295 | 52.8048 | 0.0355 | 59.8626 |
| 0.0056 | 12.3378 | 0.0116 | 24.3535 | 0.0176 | 35.1245 | 0.0236 | 44.6509 | 0.0296 | 52.9326 | 0.0356 | 59.9697 |
| 0.0057 | 12.5482 | 0.0117 | 24.5432 | 0.0177 | 35.2935 | 0.0237 | 44.7991 | 0.0297 | 53.0601 | 0.0357 | 60.0764 |
| 0.0058 | 12.7584 | 0.0118 | 24.7326 | 0.0178 | 35.4621 | 0.0238 | 44.9470 | 0.0298 | 53.1872 | 0.0358 | 60.1828 |
| 0.0059 | 12.9681 | 0.0119 | 24.9216 | 0.0179 | 35.6304 | 0.0239 | 45.0945 | 0.0299 | 53.3140 | 0.0359 | 60.2889 |

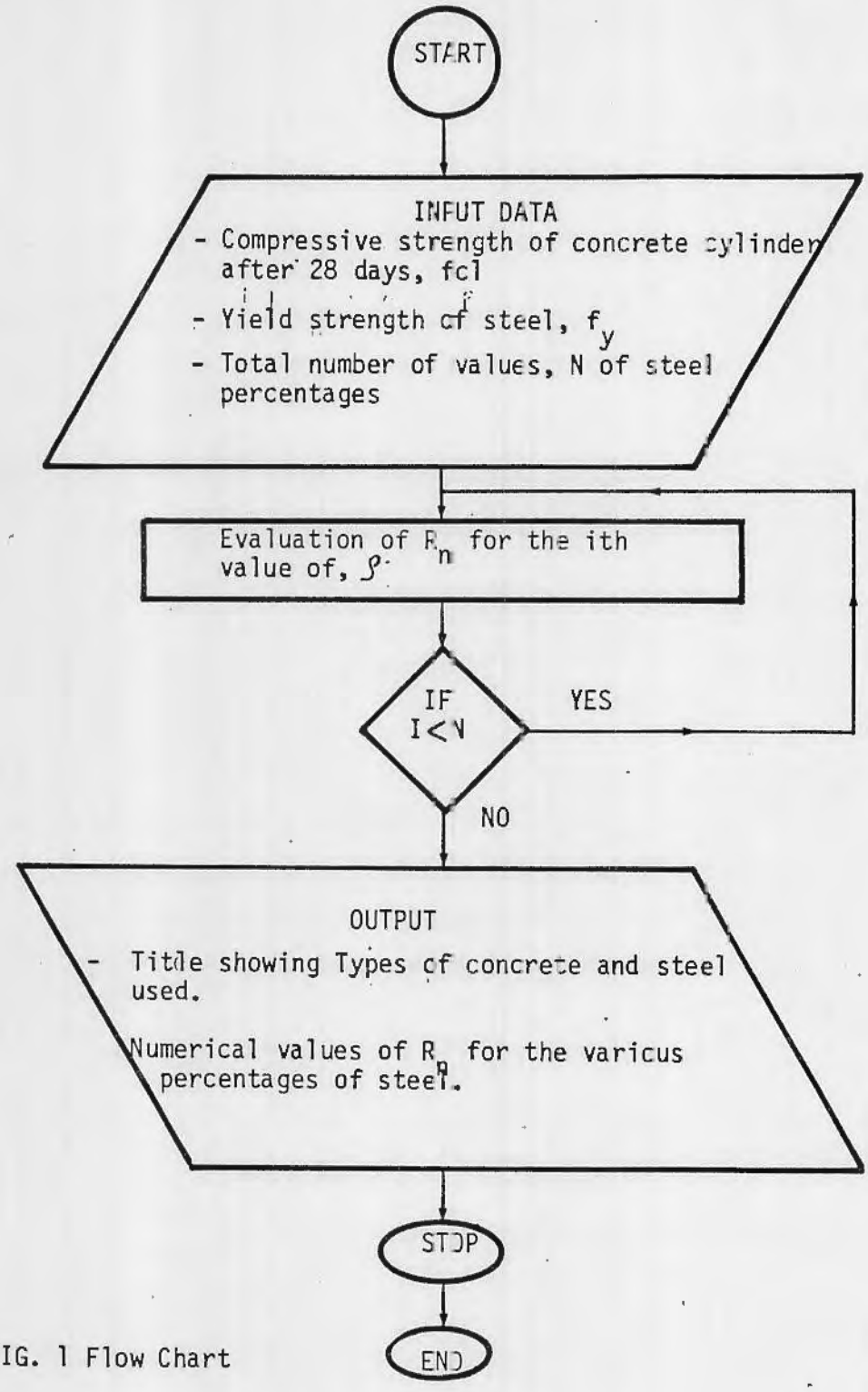


FIG. 1 Flow Chart

# COMPUTER AIDED ANALYSIS OF CONTINUOUS BEAMS BY THE EQUATION OF THREE MOMENTS

BY

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In this work, a Fortran computer program for the analysis of continuous beams by using the Equation of Three Moments is presented. The beams are assumed to have different stiffnesses and subjected to different combinations of the types of loads as shown in fig. 1.

In order to facilitate the development of the computer program, the three moments equations are represented by the following matrix,

$$\underset{NXN}{[F]} \; \underset{N}{\{M\}} = \underset{N}{\{R\}} \qquad (1)$$

where $\{M\}$- a vector containing the connecting moments at the intermediate supports.

$\{R\}$- a vector containing the elastic reactions resulting from the externally applied loads.

N - the degree of indeterminacy of the structure

$[F]$- flexibility matrix. The various elements of the matrix are as follows:-

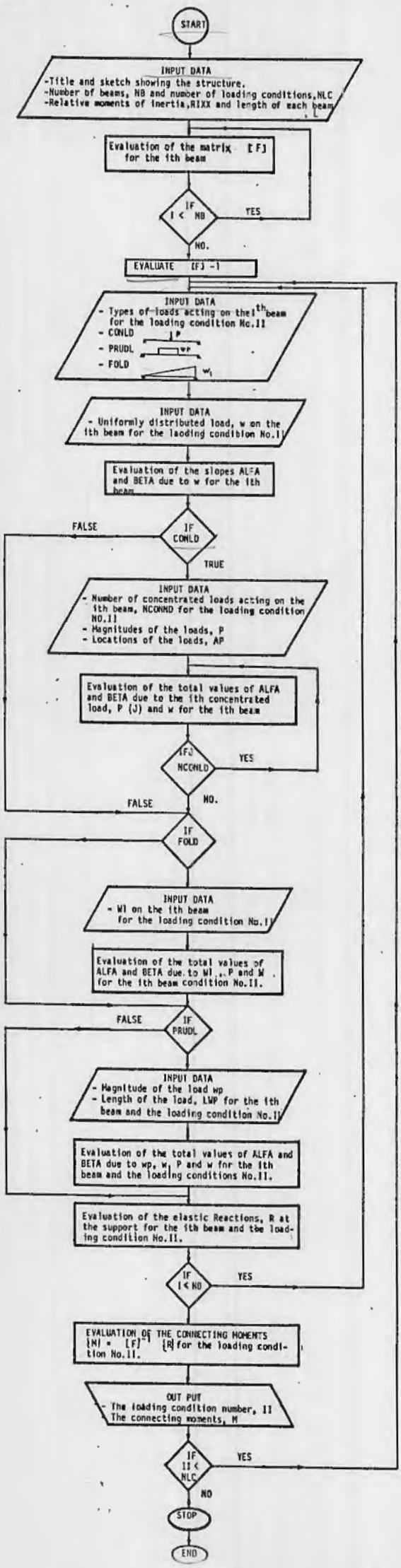
$$F_{ii}=2\left[\frac{L_i}{E_i I_i}+\frac{L_j}{E_j I_j}\right], \quad i=1,2,\ldots N \quad j=i+1 \qquad (2\text{-a})$$

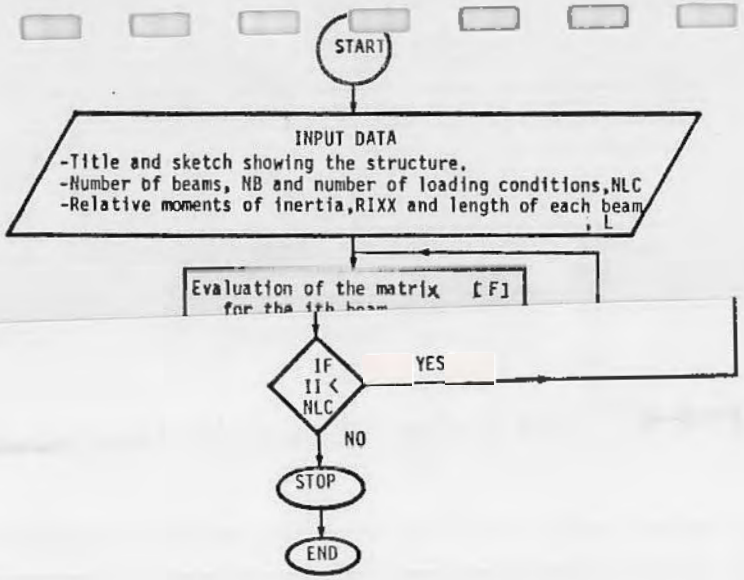
$$F_{ij}=F_{ji} \qquad (2\text{-b})$$

$$F_{ij}=\frac{L_j}{E_j I_j} \qquad (2\text{-c})$$

and all the other elements an equal to zero.

The flow chart and the listing of the computer program are shown in Figures 2 and 3, respectively. Sample outputs are presented in Figure 4.


START
INPUT DATA
-Title and sketch showing the structure.
-Number of beams, NB and number of loading conditions, NLC
-Relative moments of inertia, RIXX and length of each beam
Evaluation of the matrix [F] for the ith beam
IF I < NB
YES
NO.
EVALUATE [F] -1
INPUT DATA
- Types of loads acting on the ith beam for the loading condition No.II
- CONLD
- PRUDL
- FOLD
INPUT DATA
- Uniformly distributed load, w on the ith beam for the laoding condition No.II
Evaluation of the slopes ALFA and BETA due to w for the ith beam
FALSE
IF CONLD
TRUE
INPUT DATA
- Number of concentrated loads acting on the ith beam, NCONLD for the loading condition NO.II
- Magnitudes of the loads, P
- Locations of the loads, AP
Evaluation of the total values of ALFA and BETA due to the ith concentrated load, P (J) and w for the ith beam
IF J NCONLD
YES
NO.
FALSE
IF FOLD
INPUT DATA
- Wl on the ith beam for the loading condition No.II
Evaluation of the total values of ALFA and BETA due to Wl, P and W for the ith beam condition No.II.
FALSE
IF PRUDL
INPUT DATA
- Magnitude of the load wp
- Length of the load, LWP for the ith beam and the loading condition No.II
Evaluation of the total values of ALFA and BETA due to wp, wl P and w for the ith beam and the loading conditions No.II.
Evaluation of the elastic Reactions, R at the support for the ith beam and the loading condition No.II.
IF I < NB
YES
EVALUATION OF THE CONNECTING MOMENTS {M} = [F]-1 {R} for the loading condition No.II.
OUT PUT
- The loading condition number, II
The connecting moments, M
IF II < NLC
YES
NO
STOP
END


START
INPUT DATA
-Title and sketch showing the structure.
-Number of beams, NB and number of loading conditions,NLC
-Relative moments of inertia,RIXX and length of each beam , L
Evaluation of the matrix [F]
IF II < NLC
YES
NO
STOP
END

A FORTRAN COMPUTER PROGRAM FOR THE ANALYSIS OF CONTINUOUS BEAMS
BY USING THE EQUATION OF THREE MOMENTS
DEVELOPED BY
DR. IBRAHIM MAHFOUZ MOHAMED IBRAHIM

```
    wt/m'                 | p t          wt/m'
.............             v            .............
o-----------o-------------------o-------------o
|....4m.....|...3m...|...3m...|....4m......|
```

LOADING CONDITION NO. 1

| SPAN | 1 | 2 | 3 |
|---|---|---|---|
| W | 1.00 | .00 | 1.00 |
| W1 | .00 | .00 | .00 |
| WP | .00 | .00 | .00 |
| P | .00 | 4.00 | .00 |

CONNECTING MOMEMNTS

| INTER. SUPPORT | 1 | 2 |
|---|---|---|
| M | -2.69 | -2.69 |

LOADING CONDITION NO. 2

| SPAN | 1 | 2 | 3 |
|---|---|---|---|
| W | 2.00 | .00 | 2.00 |
| W1 | .00 | .00 | .00 |
| WP | .00 | .00 | .00 |
| P | .00 | 8.00 | .00 |

CONNECTING MOMEMNTS

| INTER. SUPPORT | 1 | 2 |
|---|---|---|
| M | -5.38 | -5.38 |

Stop - Program terminated.

A>EQ3M B:DEQ3M.FRM

A FORTRAN COMPUTER PROGRAM FOR THE ANALSIS OF FRAMES
BY USING THE EQUATION OF THREE MOMENTS
DEVELOPED BY
DR. IBRAHIM MAHFOUZ MOHAMED IBRAHIM

```
        10 T !
             !  2T/M'
  ! . . . . .V. . . . . . ..
  !-- -- -- -- -- -- -- --+ .          +
  !          EI            ! .
                           ! .
                           ! .
                     EI    ! . 2T/M'   4M
                           ! .
                           ! .
                           ! .
                           ! .
                        ==!===         +

  +    2M    +    2M     +
```

LOADING CONDITION NO. 1

| SPAN | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| W | .00 | 2.00 | 2.00 | .00 |
| W1 | .00 | .00 | .00 | .00 |
| WP | .00 | .00 | .00 | .00 |
| P | .00 | 10.00 | .00 | .00 |

CONNECTING MOMEMNTS

| INTER. SUPPORT | 1 | 2 | 3 |
|---|---|---|---|
| M | -8.92 | -5.17 | -1.42 |

# A BRIEF HISTORY OF THE DEVELOPMENT OF COMPUTER ORIENTED STRUCTURAL ANYLYSIS TECHNIQLES

BY

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Up to recently, structural engineers have to solve even the most complex problems with only a slide rules or hand calculators. As a result, long hours have been usually spent in performing the structural analysis. Often a grossly simplified model of the actual structure had to be used and the number of load conditions kept to a minimum. In addition, structural engineering judgement rather than reliable analytical data had to be used to an exceptional degree in specifying structural requirements, especially at the preliminary formative design stages.

This unsatisfactory state of affairs underwent a drastic change with the advent of the high-speed electronic computers. In seeking to exploit fully the advantages offered by electronic computers, It soon became evident that the existing crude idealizations used in approximating actual structures might be replaced by greatly refined models. This, in turn, stimulated the development of the matrix methods of structural analysis which first began appearing in the early 1950s. It was recognized that when fundamental structural principles are applied to the calculation of deformations and internal forces in structure built up from many component parts, matrices furnish the most convenient mathematical language for expressing the theory. Furthermore, the solution of the equations expressing the theory can be best achieved as a sequence of matrix numerical calculations. The computer is ideally suited to carrying out these matrix operations.

## THE MATRIX FORCE METHOD.

As might have been guessed, the first steps at providing solutions followed traditional lines, which had grown out of the theory of elasticity and followed the general approach of calculating internal stresse by using fundamental conditions of equilibrium of forces and continuity of displacements. Deformations could then be obtained by a second, subsequent calculation. This procedure is known as the redundant force method. In recent years it has been often called the force method. In the early 1950s, force method was formulated in general matrix terms and, in addition, introduced the use of the electronic computer for obtaining the desired solutions. Subsequently, it was realized that displacements, rather that internal forces, could be chosen as the primary unknowns of the structural problem. This alternate choice of unknowns then brought the stiffness matrix into the formulation. In early works the stiffness matrix was not calculated directly as in present-day finite element work, but rather was obtained by inverting the then more familiar Flexibility matrix, Hence, the stiffness method was considered as an alternate choice for solving the equations generated from the force method point of view.

## THE MATRIX STIFFNESS METHOD
## " THE DISPLACEMENT METHOD "

In spite of the foregoing progress in developing the force method, it became clear in the first half of the 1950s that the ultimate solution might well have to be sought in other directions. This was due to the fact that the force method was unable to take full advantage of the computer's capability and that the force method burdens the user with extensive data preparation (e.g. setting up and inputing the equilibrium equations). As a result, the stiffness method and subsequently the finite element or discrete element methods which are computer oriented techniques requiring minimum data preparation were developed. The stiffness and finite element methods have been developed to the point where they can be applied to almost any class of engineering structures. A partial list of structural problems which can be investigated, using these matrix procedures, is as follows,

1- Two-diminsional and space trusses and frameworks.

2- Thin plates and slabs of arbitrary shapes

3- Shell structures of arbitrary geometry

4- Composite structures, including combinations of plates, curved-shell elements, and truss and beam elements,

5- Solid bodies of irregular shapes.

# INTRODUCTION TO THE FINIT ELEMENT DISPLACEMENT METHOD OF STRUCTURAL ANALYSIS.

BY

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EGYPT

The basic idea of the finite element methods of structural analysis is to replace an actual complex structural system by a set of simpler structural elements. In formulating the simplified or idealized structural system, use is made of the so-called finite element method. If the simpler system can be solved, and the resulting solution represents the true solution with satisfactory accuracy, the finite element method would have obviously served a useful purpose.

From the preceding statement it can be realized the finite element method of analysis requires the derivation of the formulation of the basic elements commonly employed structural engineering practice. These element are as follows: -

1- Truss Element
2- Frame work element
3- In-plane plate element
4- Plate bending element
5- Axisymmetric thin & shell element
6- Curved thin shell
7- Solid elements
8- Axisymmetric soild elements

The various types of element are shown in figure 1.

In this work, and in order that important ideas and method procedures may be set forth with a minimum of confusion, the discussion will be limited to the direct stiffness " Displacement ', analysis of simple types of structural systems as listed as items 1 and 2. This includes, but not limited to, the analysis of plane and space trusses, frames and grid work.

In the displacement method of structural analysis, the formulations of the element (e,g, truss element, beam element, ... etc), are first derived with respect to the local coordinate system $\tilde{X}$, $\tilde{Y}$ and $\tilde{Z}$ of the element. It will be shown for the various types of elements considered herein that the forces and moments at the two ends of the beam element, $\left\{\tilde{P}_e\right\}$, are related to the end displacements and the end rotations of the element, $\left\{\tilde{X}_e\right\}$, by the following relation.

$$\left[\tilde{K}_e\right]\left\{\tilde{X}_e\right\} = \left\{\tilde{P}_e\right\} \tag{1}$$

where $\tilde{K}_e$ is the element stiffness matrix in the local coordinate system. In order that the geometric admissibility conditions of the principal of minimum total potential energy (i.e. compatibility of deformations and rotations) can be imposed at the common ends of the rigidly connected elements, the end displacements must be transformed from the local coordinate system of the individual element to a single reference coordinate system for the structure, X,Y,Z. The relation between the displacements and rotations in the reference coordinate system, $\left\{X_e\right\}$ and those in the local coordinate system $\left\{\tilde{X}_e\right\}$ can be expressed as: -

$$\left\{\tilde{X}_e\right\} = \left[T\right]\left\{X_e\right\} \tag{2}$$

where $\left[T\right]$ is the transformation matrix. Similarly the relation between the end forces and moments in the reference coordinate system $\left\{P_e\right\}$ and those in the local coordinate system $\left\{\tilde{P}_e\right\}$ can be written in the form.

$$\left\{\tilde{P}_e\right\} = \left[T\right]\left\{P_e\right\} \tag{3}$$

substilution of the relations 2 and 3 into Equations (1), yields.

$$\left[K_e\right]\left\{X_e\right\} = \left\{P_e\right\} \tag{4}$$

where $\left[K_e\right]$ is the element stiffness matrix in the reference coordinate system given by,

$$\left[K_e\right] = \left[T\right]^T\left[\tilde{K}_e\right]\left[T\right] \tag{5}$$

by imposing the geometric admissibility conditions between the various beams, as well as, the supporting conditions, the following relation can be written,

$$\left[K\right]\left\{X\right\} = \left\{P\right\} \tag{6}$$

Where $[K]$ is the overall stiffness matrix associated with the independent degrees of freedom residing in $\{X\}$. Solution of the preceding equation (Eq. 6), yields the displacement $\{X\}$. Subsequently, the forces and moments at the two ends of each beam can be obtained.

The flow chart of a general purpose computer program using the displacement method of structural analysis described herein and developed by the writer is shown in Fig. 1.

In the articles that will follow the displacement method formulations, that is, stiffness and transformation matrix for specific structural systems are presented. The cases considered are as follows: -

1- Plane Trusses
2- Space Trusses
3- Plane Frames
4- Shear Walls
5- Space Frames
6- Structural grid systems
   (i.e. paneled beams)

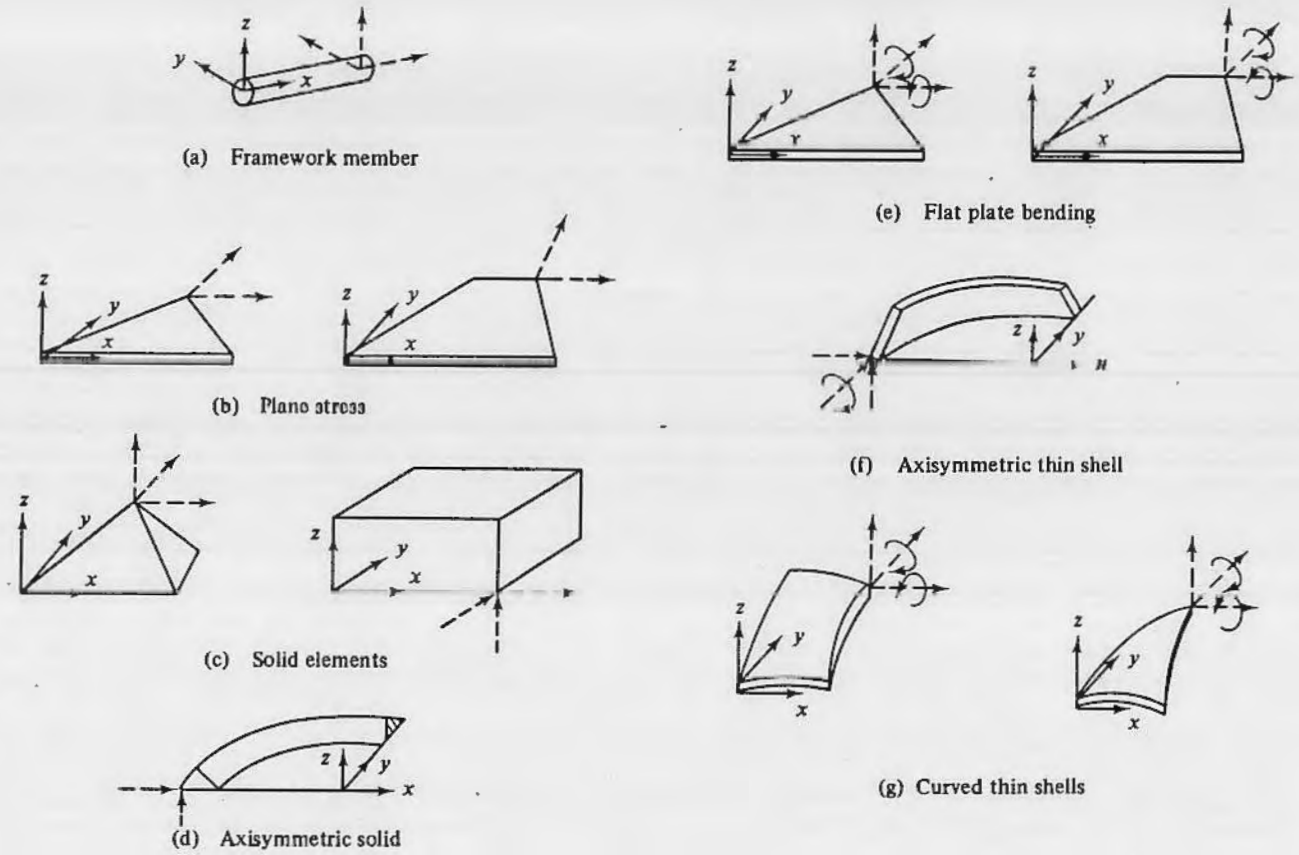


**Fig. 1** Types of finite elements.

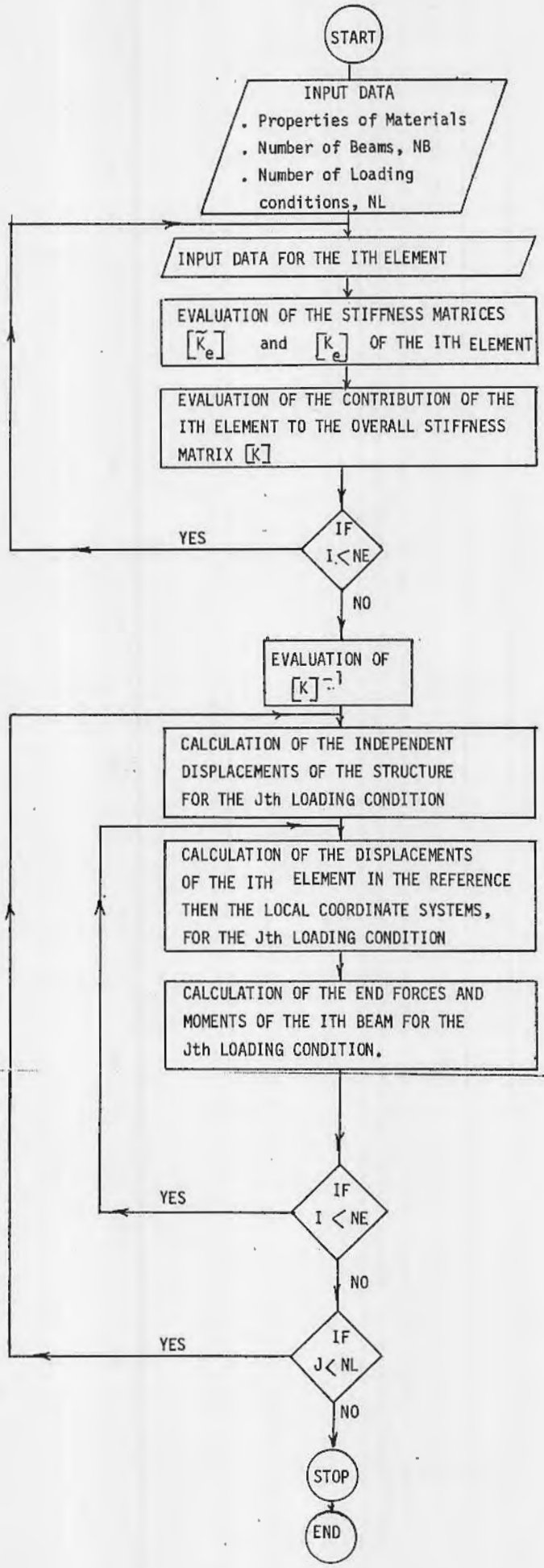


FIGURE 2. FLOW CHART OF THE COMPUTER PROGRAM

# I- ANALYSIS OF TRUSS STRUCTURES BY THE DISPLACEMENT METHOD

BY

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In this article, the displacement method of analysis of plane and space truss structures is presented. Consider the truss element shown in Figure 1. The positive directions of the axial forces and axial displacements of the two ends of the element in the local coordinate system $\tilde{X}, \tilde{Y}$ and $\tilde{Z}$ of the element are assumed to be in the directions shown in Fig. 1.

The element stiffness matrix can be derived by applying the principal of the minimum total potential energy. Alternatively the elements of the stiffness matrix can be obtained using standard structural engineering approach. The resulting element stiffness matrix in the local coordinate system $\left[\tilde{K}_e\right]$ can be expressed as: -

$$\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \tilde{U}_1 \\ \tilde{U}_2 \end{Bmatrix} = \begin{Bmatrix} \tilde{F}_1 \\ \tilde{F}_2 \end{Bmatrix} \quad (1)$$

$$\left[\tilde{K}_e\right] \quad = \quad \left\{\tilde{X}_e\right\} \quad = \quad \left\{\tilde{P}_e\right\} \quad (1b)$$

The relation between the end displacement of an element in the local and the global coordinate system shown in Fig. 2, can be represented by

$$\begin{Bmatrix} \tilde{U}_1 \\ \\ \tilde{U}_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta_x & \cos\theta_y & \cos\theta_z & 0 & 0 & 0 \\ \\ 0 & 0 & 0 & \cos\theta_x & \cos\theta_y & \cos\theta_z \end{bmatrix} \begin{Bmatrix} U_1 \\ V_1 \\ W_1 \\ U_2 \\ V_2 \\ W_2 \end{Bmatrix} \quad (2a)$$

$$\left\{\tilde{X}_e\right\} = [T] \left\{X_e\right\} \quad (2b)$$

Note that for two dimensional truss structures in the x,z plane,

$$V_1 = V_2 = 0, \text{ also, } \cos\theta_y = o$$

IMPLEMENTATION

The input data for the computer program out-lined in the preceding article using the displacement method of structural analysis, are as follows, for the cases of truss structures.

a) Title
b) Number of truss elements, degrees of freedom and loading conditions.
c) Joint loads for the various loading conditions.
d) Degrees of freedom and restrained joints
e) Cross-sectional area and the X, Y, Z coordinates of the two ends of each truss element.

Sample output of the program are presented in Figures 3-4 for the cases of plane and in Figure 5 for a space truss.

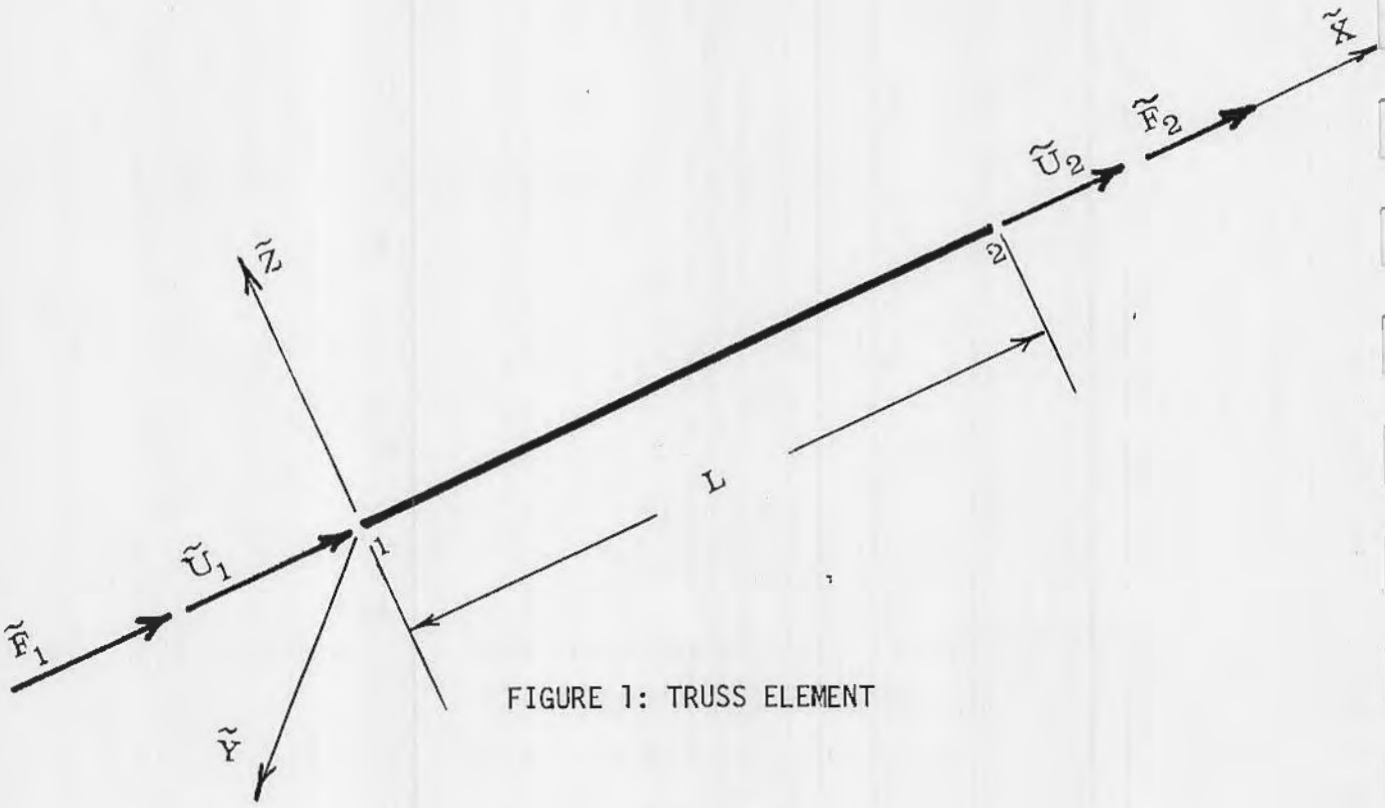


FIGURE 1: TRUSS ELEMENT

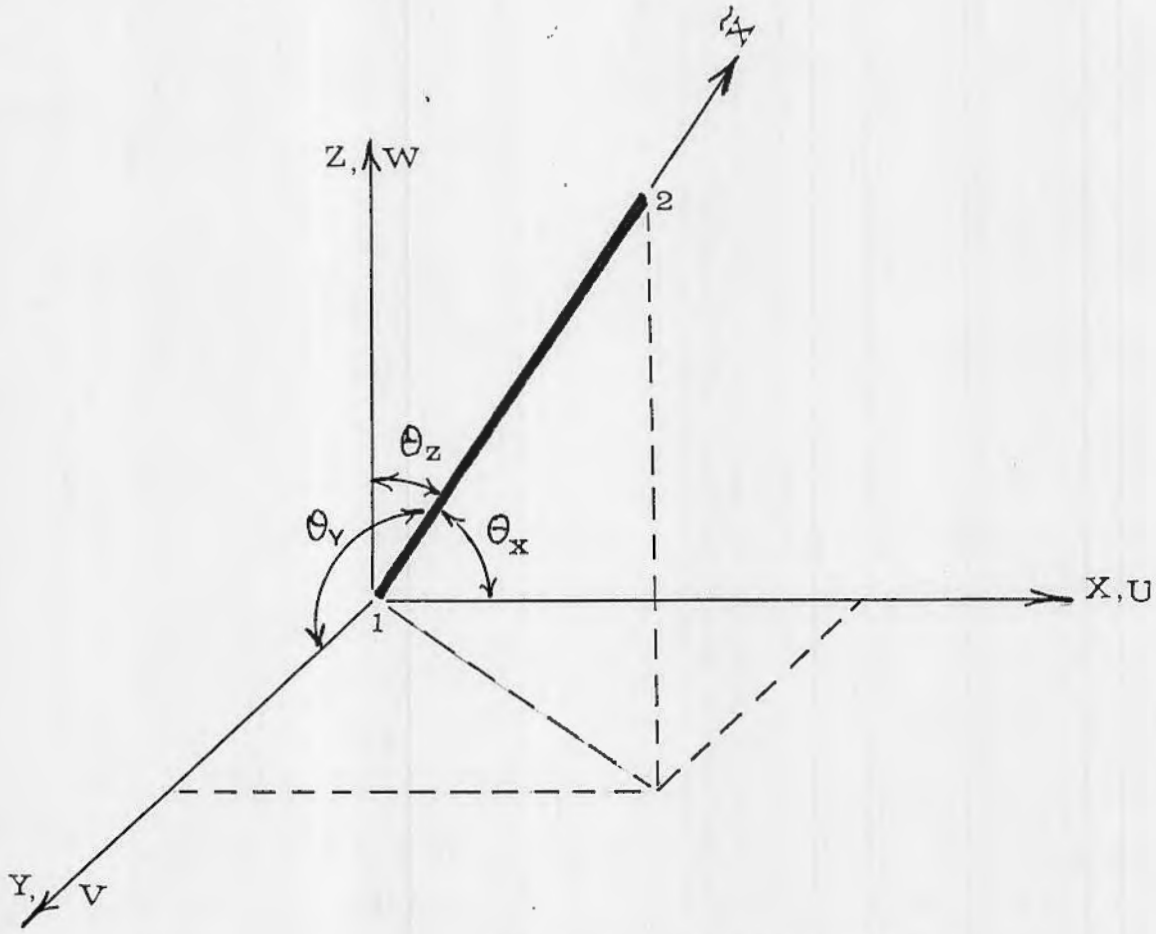


FIGURE 2: LOCAL AND REFERENCE COORDINATE SYSTEMS

| | | | | | |
|---|---|---|---|---|---|
| 2 | 0 | 3 | 4 | 0 | 0 |
| 2 | 0 | 3 | 5 | 0 | 6 |
| 4 | 0 | 0 | 5 | 0 | 6 |
| 4 | 0 | 0 | 7 | 0 | 0 |
| 5 | 0 | 6 | 7 | 0 | 0 |
| 5 | 0 | 6 | 8 | 0 | 9 |
| 7 | 0 | 0 | 8 | 0 | 9 |
| 7 | 0 | 0 | 10 | 0 | 0 |
| 8 | 0 | 9 | 10 | 0 | 0 |
| 8 | 0 | 9 | 11 | 0 | 12 |
| 10 | 0 | 0 | 11 | 0 | 12 |
| 10 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 12 | 0 | 0 | 0 |

E= 2100.000 T/CM**2

GEOMETRIC PROPERTIES OF THE TRUSS MEMBERS

| MEMBER | A | XB | YB | ZB | XE | Y E | ZE |
|---|---|---|---|---|---|---|---|
| 1 | 5.00 | .00 | .00 | .00 | 100.00 | .00 | 100.00 |
| 2 | 5.00 | .00 | .00 | .00 | 200.00 | .00 | .00 |
| 3 | 5.00 | 100.00 | .00 | 100.00 | 200.00 | .00 | .00 |
| 4 | 5.00 | 100.00 | .00 | 100.00 | 300.00 | .00 | 100.00 |
| 5 | 5.00 | 200.00 | .00 | .00 | 300.00 | .00 | 100.00 |
| 6 | 5.00 | 200.00 | .00 | .00 | 400.00 | .00 | .00 |
| 7 | 5.00 | 300.00 | .00 | 100.00 | 400.00 | .00 | .00 |
| 8 | 5.00 | 300.00 | .00 | 100.00 | 500.00 | .00 | 100.00 |
| 9 | 5.00 | 400.00 | .00 | .00 | 500.00 | .00 | 100.00 |
| 10 | 5.00 | 400.00 | .00 | .00 | 600.00 | .00 | .00 |
| 11 | 5.00 | 500.00 | .00 | 100.00 | 600.00 | .00 | .00 |
| 12 | 5.00 | 500.00 | .00 | 100.00 | 700.00 | .00 | 100.00 |
| 13 | 5.00 | 600.00 | .00 | .00 | 700.00 | .00 | 100.00 |
| 14 | 5.00 | 600.00 | .00 | .00 | 800.00 | .00 | .00 |
| 15 | 5.00 | 700.00 | .00 | 100.00 | 800.00 | .00 | .00 |

NODAL DISPLACEMENTS IN THE REFERENCE COORDINATES

| | | | | | | |
|---|---|---|---|---|---|---|
| -.286 | -.202 | -.163 | -.230 | -.162 | -.319 | -.130 |
| -.096 | -.318 | -.034 | -.050 | -.152 | | |

| MEMBER NUMBER | AXIAL FORCE TON |
|---|---|
| 1 | -4.161 |
| 2 | 2.942 |
| 3 | -9.982 |
| 4 | 2.116 |
| 5 | -13.193 |
| 6 | 5.213 |
| 7 | -15.091 |
| 8 | 3.458 |
| 9 | -14.869 |
| 10 | 5.056 |
| 11 | -13.415 |
| 12 | 2.430 |
| 13 | -8.789 |
| 14 | 1.785 |
| 15 | -5.353 |

Stop - Program terminated.

SPACE TRUSS

**********

OUTPUT OF A COMPUTER PROGRAM DEVELOPED BY

DR. IBRAHIM MAHFOUZ MOHAMED IBRAHIM

**********

```
   10 T ----->o    +
              . .
            .   .
          .     .    300
  Z     .     6 .
  | .           .
  o . . . . . . o    +    --> X
  .   2,3       .
 _o_           _o_
```

```
                    ****************************
                          PLANE TRUSS
                    ****************************
OLT PUT OF A COMPUTER PROGRAM FO THE ANALYSIS OF TRUSS STRUCTURES BY
                    USING THE DISPLACEMENT METHOD
                          DEVELOPED BY
                    DR. IBRAHIM MAHFOUZ MOHAMED IBRAHIM


       10 t          20 t          20 t          10 t
z       |             |             |             |
|       v             v             v             v
2t ---->o . . . . . . o . . . . . . o . . . . . . o          +
    .     .     .     .     .     .     .     .     .
1 .       3.    .5    7 .   .9      11.   .13     15.       1m
o . . . . . . o . . . . . . o . . . . . . o . . . . . . o    + ---> x
o     2             6             10            14      /\

|    2m       |     2m      |     2m      |     2m      |

    NE=  15     NDF=  15

    P
          .000     2.000   -10.000      .000      .000   -20.000      .000
```

# II- ANALYSIS OF PLANE AND SPACE FRAMES BY THE DISPLACEMENT METHOD

BY

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BANHA BRANCH, DEPT. OF CIVIL ENG.
EGYPT

In this article, the displacement method is extended to the analysis of plane and space frame structures.

## A- PLANE FRAME STRUCTURE

In this section the analysis of two-dimensional frames and shear wall structural systems by the displacement method is presented. It is pointed out thatformulation presented herein is general and include in addition to the flexural and axial deformations of the element, the effect of shear deformation which is quiet significant in the cases of shear wall structural systems. Consider the beam (i.e. frame work) element shown in Fig. 1. The positive directions of the forces and moments, as well as, the displacements and rotations of the two ends of the beam element in the local coordinate system of the element are assumed to be in the directions shown in Fig. 1.

The element stiffness matrix $[\tilde{K}_e]$ derived using either the principal of minimum total potential energy, or standard structural engineering principals, which includes the effects of axial, flexural and shear deformation, can be expressed as shown in Equation 1a:-

Note that the coefficient $\alpha$, accounts for the shear deformation in the deep beam element that is, the wall element and vanishes for standard slender beam element.

The transformation matrix derived by imposing the compatibility of deformations between the displacements and rotations in the local and the reference coordinate systems, can be expressed as given in relation (2b),

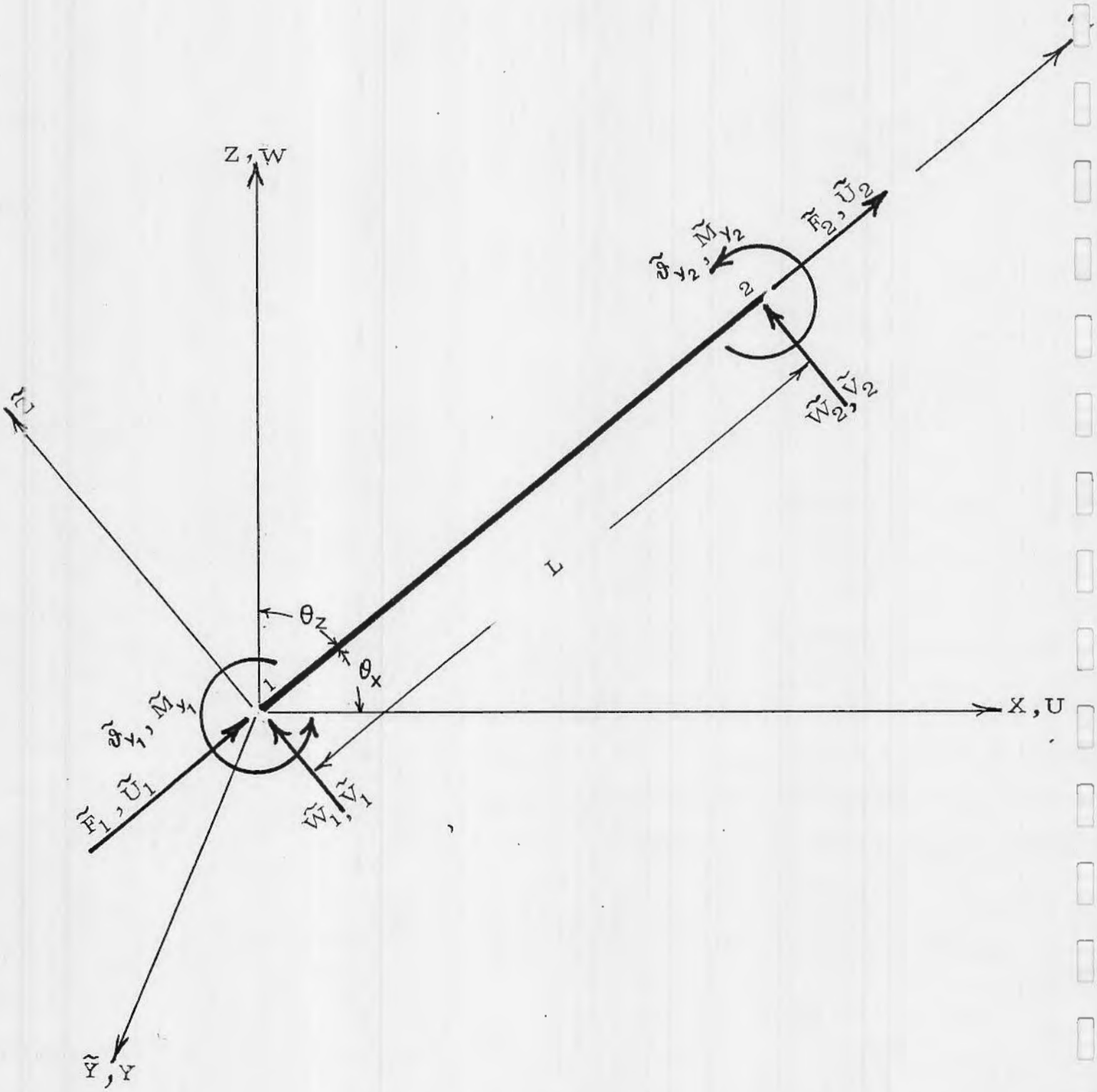


FIGURE 1: TWO DIMENSIONAL BEAM ELEMENT

$$
\begin{bmatrix}
\frac{AE}{L} & 0 & 0 & K_{1,1} & 0 & 0 \\
 & \frac{12\,EI_y}{(1+\alpha)\,L^3} & \frac{6\,EI_y}{(1+\alpha)L^2} & 0 & -K_{2,2} & K_{2,3} \\
 & & \frac{(4+\alpha)\,EI_y}{L} & 0 & -K_{2,3} & \frac{(2-\alpha)\,EI_y}{(1+\alpha)\,L} \\
 & & & K_{1,1} & 0 & 0 \\
 & \text{(SYMMETRICAL)} & & & K_{2,2} & -K_{2,3} \\
 & \alpha = \frac{12\,E\,I_y}{G\,A_r\,L^2} & & & & K_{3,3}
\end{bmatrix}
\begin{Bmatrix}
\tilde{U}_1 \\ \tilde{W}_1 \\ \tilde{\theta}_{y_1} \\ \tilde{U}_2 \\ \tilde{W}_2 \\ \tilde{\theta}_{y_2}
\end{Bmatrix}
=
\begin{Bmatrix}
\tilde{F}_1 \\ \tilde{V}_1 \\ \tilde{M}_1 \\ \tilde{F}_2 \\ \tilde{V}_2 \\ \tilde{M}_2
\end{Bmatrix}
$$

$$
\left[\tilde{K}_e\right] \qquad \left\{\tilde{X}_e\right\} = \left\{\tilde{P}_e\right\}
$$

$$
\underbrace{\left\{\begin{array}{c}
\tilde{U}_1 \\
\tilde{W}_1 \\
\tilde{\theta}_{y_1} \\
\hline
\tilde{U}_2 \\
\tilde{W}_2 \\
\tilde{\theta}_{y_2}
\end{array}\right\}}_{\{\tilde{X}_e\}}
=
\underbrace{\left[\begin{array}{ccc|ccc}
\cos\theta_x & \cos\theta_z & 0 & 0 & 0 & 0 \\
-\cos\theta_z & \cos\theta_x & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & \cos\theta_x & \cos\theta_z & 0 \\
0 & 0 & 0 & -\cos\theta_z & \cos\theta_x & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]}_{[T]}
\underbrace{\left\{\begin{array}{c}
U \\
W_1 \\
\theta_{y_1} \\
\hline
U_2 \\
W_2 \\
\theta_{y_2}
\end{array}\right\}}_{\{X_e\}}
\qquad (2b)
$$

## B- SPACE FRAME STRUCTURES

The discussion in this section is focused on the analysis of three dimensional frame structural systems by the displacement method. The formulation presented herein can be used in the analysis of a wide variety of three dimensional structural systems, such as paneled beam floors, general space frame structures, raft foundations idealized as structural grid system, and three dimensional shear wall core systems.

The element stiffness matrix $[\tilde{K}e]$ of a three dimensional beam element (Fig.2) in the local coordinate system which includes the effects of axial, torsional, unsymetrical flexural and shear deformations, can be expressed as shown in Equation 3a. The relation between the end displacements of a three dimensional beam element in the local and the reference (global) coordinate systems can be expressed by the matrix relation 4a.

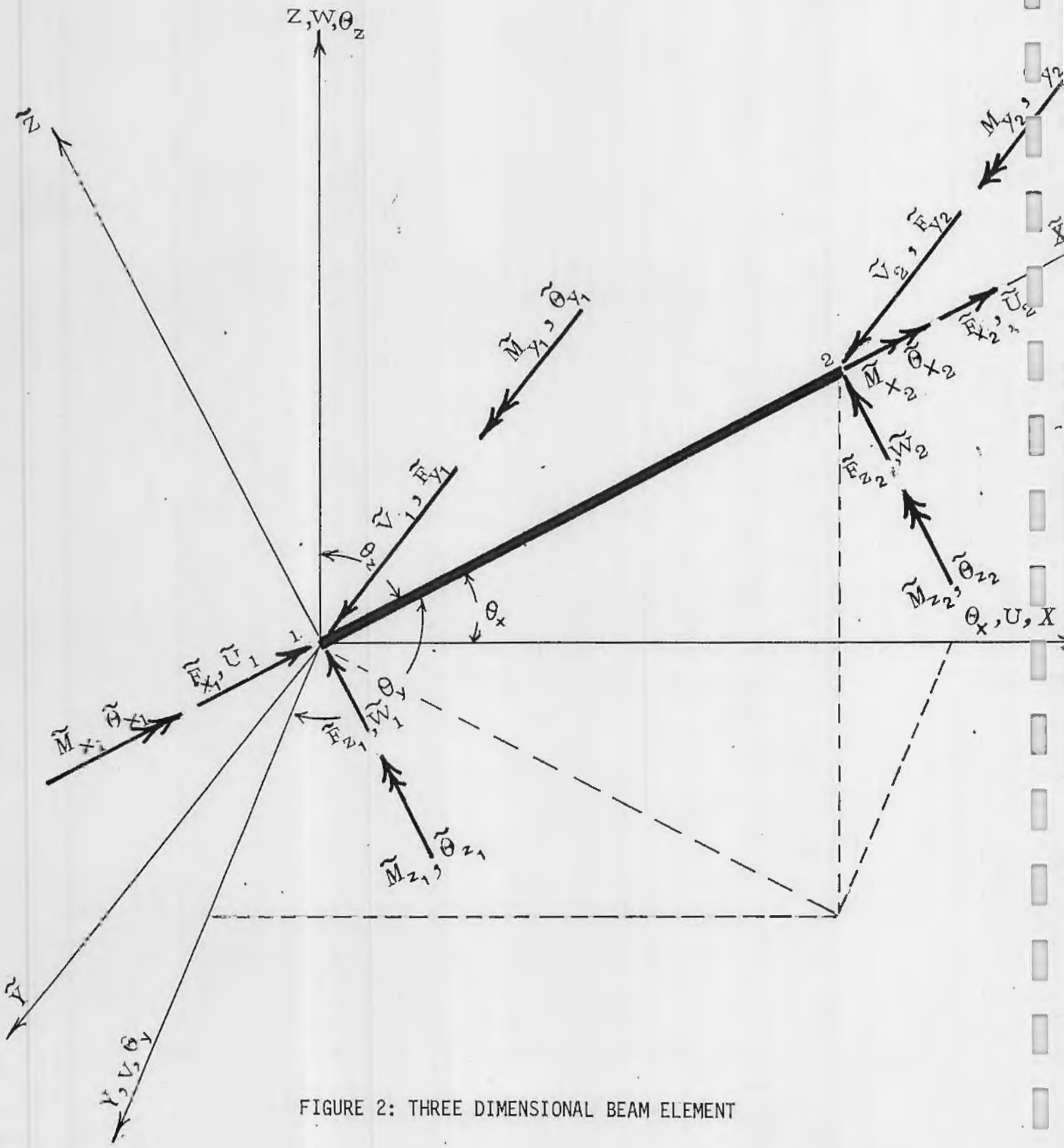


FIGURE 2: THREE DIMENSIONAL BEAM ELEMENT

$$
\begin{bmatrix}
\frac{AE}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
 & \frac{12\,EI_z}{[1+\alpha_z]L^3} & 0 & 0 & 0 & -\frac{6EI_z}{[1+\alpha_z]L^2} & 0 & -\frac{12\,EI_z}{[1+\alpha_z]L^3} & 0 & 0 & 0 & -\frac{6\,EI_z}{[1+\alpha_z]L^2} \\
 & & \frac{12\,EI_y}{[1+\alpha_y]L^3} & 0 & \frac{6\,EI_y}{[1+\alpha_y]L^2} & 0 & 0 & 0 & -\frac{12\,EI_y}{[1+\alpha_y]L^3} & 0 & \frac{6\,EI_y}{[1+\alpha_y]L^2} & 0 \\
 & & & \frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
 & & & & \frac{[4+\alpha_y]EI_y}{[1+\alpha_y]L} & 0 & 0 & 0 & -\frac{6\,EI_y}{[1+\alpha_y]L^2} & 0 & \frac{[2-\alpha_y]\,EI_y}{[1+\alpha_y]\,L} & 0 \\
 & & & & & \frac{[4+\alpha_z]\,EI_z}{[1+\alpha_z]\,L} & 0 & +\frac{6EI_z}{[1+\alpha_z]L^2} & 0 & 0 & 0 & \frac{[2-\alpha_z]EI_z}{[1+\alpha_z]L} \\
 & & & & & & \frac{AE}{L} & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & \frac{12\,EI_z}{[1+\alpha_z]L^3} & 0 & 0 & 0 & \frac{6\,EI_z}{[1+\alpha_z]L^2} \\
 & & \text{(Symmetrical)} & & & & & & \frac{12\,EI_y}{[1+\alpha_y]L^3} & 0 & \frac{-6EI_y}{[1+\alpha_y]L^2} & 0 \\
 & & & & & & & & & \frac{GJ}{L} & 0 & 0 \\
 & & & & & & & & & & \frac{[4+\alpha_y]EI_y}{[1+\alpha_y]\,L} & 0 \\
 & & & & & & & & & & & \frac{[4+\alpha_z]\,EI_z}{[1+\alpha_z]\,L}
\end{bmatrix}
\begin{Bmatrix}
\tilde{U}_1 \\ \tilde{V}_1 \\ \tilde{W}_1 \\ \tilde{\theta}_{x_1} \\ \tilde{\theta}_{y_1} \\ \tilde{\theta}_{z_1} \\ \tilde{U}_2 \\ \tilde{V}_2 \\ \tilde{W}_2 \\ \tilde{\theta}_{x_2} \\ \tilde{\theta}_{y_2} \\ \tilde{\theta}_{z_1}
\end{Bmatrix}
=
\begin{Bmatrix}
\tilde{F}_{x_1} \\ \tilde{F}_{y_1} \\ \tilde{F}_{z_1} \\ \tilde{M}_{t_1} \\ \tilde{M}_{y_1} \\ \tilde{M}_{z_1} \\ \tilde{F}_{x_2} \\ \tilde{F}_{y_2} \\ \tilde{F}_{z_2} \\ \tilde{M}_{t_2} \\ \tilde{M}_{y_2} \\ \tilde{M}_{z_2}
\end{Bmatrix}
\quad (3a)
$$

$$
\begin{Bmatrix} \tilde{U}_1 \\ \tilde{V}_1 \\ \tilde{W}_1 \\ \hline \tilde{\theta}_{x_1} \\ \tilde{\theta}_{y_1} \\ \tilde{\theta}_{z_1} \\ \hline \tilde{U}_2 \\ \tilde{V}_2 \\ \tilde{W}_2 \\ \hline \tilde{\theta}_{x_2} \\ \tilde{\theta}_{y_2} \\ \tilde{\theta}_{z_2} \end{Bmatrix}
=
\left[\begin{array}{c:c:c:c}
[T_1] & 0 & 0 & 0 \\ \hdashline
0 & [T_1] & 0 & 0 \\ \hdashline
0 & 0 & [T_1] & 0 \\ \hdashline
0 & 0 & 0 & [T_1]
\end{array}\right]
\begin{Bmatrix} U_1 \\ V_1 \\ W_1 \\ \hline \theta_{x_1} \\ \theta_{y_1} \\ \theta_{z_1} \\ \hline U_2 \\ V_2 \\ W_2 \\ \hline \theta_{x_2} \\ \theta_{y_2} \\ \theta_{z_2} \end{Bmatrix}
$$

$$\{\tilde{X}_e\} \qquad [T] \qquad \{X_e\} \qquad 4b$$

where

$$\left[T_1\right] = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \tag{5}$$

where

$$\left.\begin{aligned} l_i &= \cos\theta_x \\ m_i &= \cos\theta_y \\ n_i &= \cos\theta_z \end{aligned}\right\} \quad i = 1,2,3 \text{ , for the } \tilde{x},\tilde{y},\tilde{z}, \text{ respectively}$$

IMPLEMENTATION:

The input data for the computer program developed by the writer and outlined in a preceding article using the displacement method are as follows, for the various cases of two and three dimensional frame structures.

1. Titles and drawing of the structure.
2. Elastic properties of the materials.
3. Number of beams and number of loading conditions.
4. Data for each individual beam element, as follows:
   a- cross-sectional dimensions.
   b- x,y and z coordinates of the two ends of the elements. Alternatively, length of the beam element and the angle of orientation with respect to the reference coordinate axes.
   c- Types of connections at the two ends of the elements including supporting conditions.
   d- Types, magnitudes and locations of the loads, for all the loading conditions considered

sample outputs for two dimensional frame, paneled frame, paneled beam floor are presented in figures 2, 3 and 4 respectively.

SPACE FRAMES BY USING THE FINITE ELEMENT DISPLACEMENT MET-OD
DEVELOPED BY
DR. IBRAHIM MAHFOUZ MOHAMED IBRAHIM

```
        +..5M..+..5M..+..5M..+..5M..+

                            3       8
  +     O . . .O. . . O . . .O . . .O
  .     .       .       .       .       .
 5M     .       .       .1      .6      .11
  .     .       .       .       .       .
  +     O . . . . . . . . . . . . . . O
  .     .       .       .  4    .  9    .
 5M     .       .       .       .       .
  .     .       .       .2      .7      .12
  +     O . . . . . . . . . . . . . . O . C-L
  .     .       .       .  5    .  10   .
 5M     .       .       .       .       .
  .     .       .       .       .       .
  +     O . . . . . . . . . . . . . . O
  .     .       .       .       .       .
 5M     .       .       .       .       .
  .     .       .       .       .       .
  +     O . . .O. . . O . . .O . . .O
                        .
                       C-L
```

PANELED BEAM FLOOR

| MEMBER | NORMAL FORCES TON | SHEARING FORCES YTLD TON | | SHEARING FORCES ZTLD TON | |
|---|---|---|---|---|---|
| | | END 1 | END 2 | END 1 | END 2 |
| 1 | .00000E+00 | .00000E+00 | .00000E+00 | .66471E+01 | .41471E+01 |
| 2 | .00000E+00 | .00000E+00 | .00000E+00 | .25000E+01 | -.18477E-04 |
| 3 | .00000E+00 | .00000E+00 | .00000E+00 | .23215E+01 | -.26785E+01 |
| 4 | .00000E+00 | .00000E+00 | .00000E+00 | .16471E+01 | -.33529E+01 |
| 5 | .00000E+00 | .00000E+00 | .00000E+00 | .70333E-05 | -.25000E+01 |
| 6 | .00000E+00 | .00000E+00 | .00000E+00 | .83529E+01 | .33529E+01 |
| 7 | .00000E+00 | .00000E+00 | .00000E+00 | .33529E+01 | -.16471E+01 |
| 8 | .00000E+00 | .00000E+00 | .00000E+00 | .30363E+01 | -.19637E+01 |
| 9 | .00000E+00 | .00000E+00 | .00000E+00 | -.33529E+01 | -.83529E+01 |
| 10 | .00000E+00 | .00000E+00 | .00000E+00 | -.41471E+01 | -.66471E+01 |
| 11 | .00000E+00 | .00000E+00 | .00000E+00 | .19637E+01 | -.30363E+01 |
| 12 | .00000E+00 | .00000E+00 | .00000E+00 | .26785E+01 | -.23215E+01 |

| MEMBER | TWISTING MOMENTS M.T. | | BENDING MOMENTS ABOUT Y-Y M.T. | | BENDING MOMENTS ABOUT Z-Z M.T. | |
|---|---|---|---|---|---|---|
| | END1 | END2 | END1 | END2 | END1 | END2 |
| 1 | .00000E+00 | .00000E+00 | -.72533E-03 | .26985E+02 | .00000E+00 | .00000E+0 |
| 2 | .00000E+00 | .00000E+00 | .26984E+02 | .33234E+02 | .00000E+00 | .00000E+00 |
| 3 | .70938E-03 | -.70938E-03 | -.17858E+01 | -.26784E+01 | .00000E+00 | .00000E+00 |
| 4 | .40754E-03 | -.40754E-03 | .33527E+02 | .29263E+02 | .00000E+00 | .00000E+0 |
| 5 | .00000E+00 | .00000E+00 | .33234E+02 | .26984E+02 | .00000E+00 | .00000E+0 |
| 6 | -.11978E-02 | .11978E-02 | -.10812E-02 | .29263E+02 | .00000E+00 | .00000E+00 |
| 7 | -.40754E-03 | .40754E-03 | .29263E+02 | .33527E+02 | .00000E+00 | .00000E+0 |
| 8 | .17639E-02 | -.17639E-02 | -.26796E+01 | .17639E-02 | .00000E+00 | .00000E+0 |
| 9 | .11978E-02 | -.11978E-02 | .29263E+02 | -.10767E-02 | .00000E+00 | .00000E+00 |
| 10 | .00000E+00 | .00000E+00 | .26985E+02 | -.72845E-03 | .00000E+00 | .00000E+0 |
| 11 | -.17639E-02 | .17639E-02 | .17639E-02 | -.26796E+01 | .00000E+00 | .00000E+0 |
| 12 | -.70938E-03 | .70938E-03 | -.26784E+01 | -.17858E+01 | .00000E+00 | .00000E+00 |

Stop - Program terminated.

RESULTS OF A FORTRAN COMPUTER PROGRAMS FOR THE ANAYSIS OF GENERAL SPACE FRAMES BY USING THE FINITE ELEMENT DISPLACEMENT METHOD

DEVELOPED BY

DR. IERAHIM MAHFOUZ MOHAMED IBRAHIM

```
        +..5M..+..5M..+..5M..+..5M..+

                            3       8
  +     O . . .O. . . O . . .O . . .O
  .     .       .       .       .       .
 5M     .       .       .1      .6      .11
  .     .       .       .       .       .
  +     O . . . . . . . . . . . . . . O
  .     .       .       .  4    .  9    .
 5M     .       .       .       .       .
  .     .       .       .2      .7      .12
  +     O . . . . . . . . . . . . . . O . C-L
  .     .       .       .  5    .  10   .
 5M     .       .       .       .       .
  .     .       .       .       .       .
  +     O . . . . . . . . . . . . . . O
  .     .       .       .       .       .
 5M     .       .       .       .       .
  .     .       .       .       .       .
  +     O . . .O. . . O . . .O . . .O
                        .
                       C-L
```

PANELED BEAM FLOOR

| MEMBER | NORMAL FORCES TON | SHEARING FORCES YTLD TON | | SHEARING FORCES ZTL TON | |
|---|---|---|---|---|---|
| | | END 1 | END 2 | END 1 | END 2 |
| 1 | .00000E+00 | .00000E+00 | .00000E+00 | .66471E+01 | .41471E+01 |
| 2 | .00000E+00 | .00000E+00 | .00000E+00 | .25000E+01 | -.18477E-04 |
| 3 | .00000E+00 | .00000E+00 | .00000E+00 | .23215E+01 | -.26785E+01 |
| 4 | .00000E+00 | .00000E+00 | .00000E+00 | .16471E+01 | -.33529E+01 |
| 5 | .00000E+00 | .00000E+00 | .00000E+00 | .70333E-05 | -.25000E+01 |
| 6 | .00000E+00 | .00000E+00 | .00000E+00 | .83529E+01 | .33529E+01 |
| 7 | .00000E+00 | .00000E+00 | .00000E+00 | .33529E+01 | -.16471E+01 |
| 8 | .00000E+00 | .00000E+00 | .00000E+00 | .30363E+01 | -.19637E+01 |
| 9 | .00000E+00 | .00000E+00 | .00000E+00 | -.33529E+01 | -.83529E+01 |
| 10 | .00000E+00 | .00000E+00 | .00000E+00 | -.41471E+01 | -.66471E+01 |
| 11 | .00000E+00 | .00000E+00 | .00000E+00 | .19637E+01 | -.30363E+01 |
| 12 | .00000E+00 | .00000E+00 | .00000E+00 | .26785E+01 | -.23215E+01 |

| MEMBER | TWISTING MOMENTS M.T. | | BENDING MOMENTS ABOUT Y-Y M.T. | | BENDING MOMENTS ABOUT Z-Z M.T. | |
|---|---|---|---|---|---|---|
| | END1 | END2 | END1 | END2 | END1 | END2 |
| 1 | .00000E+00 | .00000E+00 | -.72533E-03 | .26985E+02 | .00000E+00 | .00000E+00 |
| 2 | .00000E+00 | .00000E+00 | .26984E-02 | .33234E+02 | .00000E+00 | .00000E+00 |
| 3 | .70938E-03 | -.70938E-03 | -.17858E-01 | -.26784E+01 | .00000E+00 | .00000E+0 |
| 4 | .40754E-03 | -.40754E-03 | .33527E-02 | .29263E+02 | .00000E+00 | .00000E+0 |
| 5 | .00000E+00 | .00000E+00 | .33234E-02 | .26984E+02 | .00000E+00 | .00000E+00 |
| 6 | -.11978E-02 | .11978E-02 | -.10812E-02 | .29263E+02 | .00000E+00 | .00000E+0 |
| 7 | -.40754E-03 | .40754E-03 | .29263E-02 | .33527E+02 | .00000E+00 | .00000E+0 |
| 8 | .17639E-02 | -.17639E-02 | -.26796E-01 | .17639E-02 | .00000E+00 | .00000E+00 |
| 9 | .11978E-02 | -.11978E-02 | .29263E-02 | -.10767E-02 | .00000E+00 | .00000E+0 |
| 10 | .00000E+00 | .00000E+00 | .26985E-02 | -.72845E-03 | .00000E+00 | .00000E-0 |
| 11 | -.17639E-02 | .17639E-02 | .17639E-02 | -.26796E+01 | .00000E+00 | .00000E+00 |
| 12 | -.70938E-03 | .70938E-03 | -.26784E-01 | -.17858E+01 | .00000E+00 | .00000E-00 |

Stop - Program terminated

# CRITERIA FOR THE EVALUATION OF COMPUTER SOFTWARE

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**The use of computers in many engineering applications has grown rapidly, especially with the recent steadily decreasing cost of computer in general and micro computers in particular.**

**Once an engineer makes the initial decision to use computer aids for a given task, many subsequent decisions become necessary. A decision must be made as to whether or not to use the available software, or to develop one's own programs. This decision must be based upon whether or not there exists software which satisfies the engineer's needs in terms of analytical or design capability and whether the software is not only available, but also economical to buy and to use. Assuming that an individual has decided to use commercial software, and has been successful in locating several programs which satisfy the engineer's need, how does he then decide which program to use? The decision to use one program instead of another can often be rationalized by examining the programs with specific evaluation criteria in mind. Such criteria can be summarized as follows:-**

**1- Compatibility between the software and the type of computer in use.**
**2- Documentation.**
**3- Input.**
**4- Solution.**
**5- Output.**

## 1- COMPATIBILITY

Before purchasing software, one must make sure that the software under consideration is compatible with the type of computer in use, in terms of the operating system (e.g. Dos, UCSD, CpM, ...etc), the language (e.g. Basic, FORTRAN, Pascal, ...etc), and that the computer has the memory / storage capacities required to support the software.

## 2- DOCUMENTATION

The effectiveness of the user's manual, and other documentation in preparing a user to deal with a problem and to understand the output, is one of the fundamental downfalls of some otherwise very useful programs. Since this is such an important problem, it requires the consideration of several sub-criteria.

### 2-a Explains Methods

The level at which the documentation outlines the techniques and the method of design used in the program varies greatly among commercially available programs. Thus, when little information is available about how a design is produced, the engineer must be wary of the results obtained.

### 2-b Data Preparation Aid

As a bare minimum, the documentation for a program should clearly explain how to prepare the input data for a program. Since it has been found, time and again, that it is in the preparation of data that a

COMMERCIAL PROGRAM EVALUATION

| Program \ Criteria | DOCUMENTATION | INPUT | SOLUTION | OUTPUT | | TOTAL EVALUATION |
|---|---|---|---|---|---|---|
| Concrete Bldg. Design | | | | | | |
| AMECO MA | ○ | ● | ● | ◑ | | ◑ |
| PCA/ESTIM MA | ◑ | ◑ | ○ | ◑ | | ◑ |
| STRUDL II SDL | ◑ | ● | ○ | ○ | | ○ |
| Concrete Slab-Beam Design | | | | | | |
| PCA/ADOSS MA | ● | ◑ | ● | ○ | | ◑ |
| SP/CBULT CDC | ◑ | ○ | ● | ○ | | ◑ |
| SP/NONPU CDC | ◑ | ○ | ● | ○ | | ◑ |
| Concrete Col. Design | | | | | | |
| PCA/COL MA | ● | ○ | ● | ○ | | ◑ |
| PCA/LS MA | ● | ◑ | ● | ● | | ● |
| Concrete Wall Design | | | | | | |
| SP/RETWAL CDC | ○ | ◑ | ○ | ○ | | ◑ |
| SP/EQPIER CDC | ○ | ○ | ○ | ○ | | ○ |
| SP/BRGWAL CDC | ○ | ○ | ◑ | ○ | | ○ |
| Concrete Fnd. Design | | | | | | |
| PCA/MAT MA | ● | ○ | ◑ | ○ | | ◑ |
| SP/COMFTG CDC | ○ | ○ | ○ | ○ | | ○ |

Evaluation Code: ● Good, ◑ Fair, ○ Poor.

| Program \ Criteria | DOCUMENTATION | INPUT | SOLUTION | OUTPUT | | TOTAL EVALUATION |
|---|---|---|---|---|---|---|
| Frame Analysis | | | | | | |
| AMECO MA | ◑ | ● | ◑ | ● | | ● |
| SAGS CDC | ● | ○ | ○ | ● | | ◑ |
| STRESS 3.0 MA | ◑ | ● | ○ | ● | | ● |
| STRUDL II SDL | ◑ | ○ | ◑ | ● | | ◑ |
| GENSAP CDC | ● | ○ | ◑ | ● | | ◑ |
| 2DGENFRAME CDC | ● | ○ | ○ | ● | | ◑ |
| Frame-Shearwall Analysis | | | | | | |
| PCA/STMFR SDL | ○ | ◑ | ◑ | ◑ | | ○ |
| SP/SHWINT CDC | ◑ | ○ | ○ | ◑ | | ◑ |
| Dynamic Frame Analysis | | | | | | |
| DAGS CDC | ● | ○ | ● | ◑ | | ◑ |
| STRUDL II SDL | ◑ | ○ | ◑ | ● | | ◑ |
| 2DFMAP CDC | ● | ○ | ○ | ● | | ◑ |

Evaluation Code: ● Good, ◑ Fair, ○ Poor.

great deal of engineers' efforts are spended.

## 2-c Output Interpretation

The manual should help the user understand the output both in case of successful and unsuccessful runs. There is little that is more frustrating than having a run return with an abundance of error messages and no real explanation of the problem.

## 2-d Honesty

There is often a gap between the **claims** of the documentation, and the **real** requirements and **capabilities** of the program. This difference may occur because the developer exaggerated the abilities of his program, or because, somewhere along the line a portion of a program became nonfuctional.

## 3- Input

The form of the input requirements can have some importance in making the program easy to use. Also, the ability of a program to correct simple errors in input data, and to provide optional default values for common data is a boon to the user, and potential timesaver. In addition, much time is often spent on generating input which could otherwise, be more efficiently used. The advantages, in terms of time saved, of a program with the ability to abbreviate the input data by having some data generated automatically, are substantial.

## 4- Solution

It is important to assess programs (Software), in terms of the analysis and / or design produced by the programs. The level of confidence which an engineer can have in an automatic design, and the ease with which an engineer exercises direct control over the design, are both factors which bear directly on how useful a specific program is to an engineer. This criterion also divides into two important sub-criteria.

## 4-A Control

An engineer does not use computer aids for analysis and / or design to avoid becoming involved in the analysis and / or design. Rather, most engineers would not feel comfortable using an automatic design procedure unless some control could be exercised over the design produced. The designer will at least want to specify such things as the design code to be invoked.

## 4-B Quality

In general terms, one should identify the degree to which the output may be considered as a final design, with consideration given to reliability of design values; satisfaction of code requirements; conformity to the most up to date design philosophy; economy of design and the presentation of alternative designs.

## 5- Output

The effectiveness of the program output in communicating the information generated by the program, in a manner useful to the engineer, must receive due consideration. In selecting a program for use, it is worthwhile to scrutinize the output of the program for clarity and organization.

The technique presented herein has been used to evaluate a sampling of commercially available programs in various categories of structural analysis and design. The results of the evaluation are summarized in the tables.

| Program \ Criteria | DOCUMENTATION | INPUT | SOLUTION | OUTPUT | | TOTAL EVALUATION |
|---|---|---|---|---|---|---|
| Steel Bldg. Design | | | | | | |
| AMECO MA | ○ | ● | ● | ● | | ● |
| STRUDL II SDL' | ● | ◑ | ● | ● | | ◑ |
| Steel Floor & Beam Design | | | | | | |
| CISC/FLOOR MA | ● | ◑ | ◑ | ○ | | ◑ |
| SP/STLBMA CDC | ○ | ○ | ● | ○ | | ◑ |
| SP/CDMPBU CDC | ○ | ○ | ● | ○ | | ○ |
| Steel Column Design | | | | | | |
| CISC/COL MA | ● | ○ | ◑ | ○ | | ◑ |
| SP/STLCOL CDC | ○ | ○ | ● | ○ | | ○ |

Evaluation Code: ● Good, ◑ Fair, ○ Poor.

# SYNOPSIS:

## SUBJECT OF THE ISSUE:-

«An Urban Study of Zamalek District», the Study points out to the necessity of putting building regulations for the area in order to preserve its character. The study was conducted by a research team from the Urban Planning Organization and presented by arch. Kamal S. Shouhaib.

## PERSONALITY OF THE ISSUE:-

It is Prof. Youssef Hassan Shafik, prof. of architectural design at Cairo University, where he started his academic career in 1942 and uptill now. He received his M.A. degree in architecture in 1953 from Illinois University, U.S.A. Prof. Y.H. Shafik participated in the design and planning of many architectural and urban planning projects in Egypt and abroad through his own consultancy office.

## TECHNICAL ARCTICLE:-

+ «Traditional Housing Design in Arab Countries», written by arch. Samir Abulac. The material presented in this study is based on some comparative approaches dealing with the following topics: ecological analysis of human settlements, analysis of traditional courtyard designs, analysis of traditional courtyard housing models, and some examples of contemporary development and disruptions.

+ «Koranic Notions: Function and Beauty in Architectural and Urban Design», written by Dr Hazem Ibrahim.

## PROJECTS OF THE ISSUE:-

+ Mixed Development at Cdhams Walk, Covent Garden: design team, D. Ball, M. O'connor, J. Watts, S. Groak. The original brief was issued by GLC in 1974 for a mixed use development. The project which lies on an area of 0.6 ha. includes 102 dwellings at a density of 472 P/ha, and 5000m$^2$ of non-housing in the form of shops, workshops, and community uses.

+ Urban Housing Projects in Algeria: arch. Abdelrahman El-Miniawy. An illustration of two urban housing projects in Algeria, in Eiskia and M'isala.

+ Private House at Jerusalem: architect Moshie Safadie. The project is a redevelopment of the ruin of an old house in Yafa.

# AUTOMATED MINIMUM COST DESIGN OF PREFABRICATED PRE-STRESSED CONCRETE BEAMS

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One of the main problems of structural designers is to select the best design for a certain structure which satisfies both structural and economical requirements, especially for prefabricated structures where the elements of the structures are usually constructed in large numbers. The objective of this work is to present a structural synthesis capability to find the optimum design of prefabricated prestressed concrete beams of various shapes automatically. The present structural synthesis concept is an inequality constrained minimization approach which is based on a number of considerations, namely, a set of design variables, an objective function, a set of constraints and a powerful mathematical programming procedure. The design variables are the independent cross-sectional dimensions of the concrete, the prestressing force, the area of prestressing steel in each row and their distances from the bottom fiber of the beam, as shown in Fig.1. The goal of the structural synthesis concept is to select these variables such that an acceptable minimum cost design is obtained. The objective function, C(V) which reflects the cost of the prestressed concrete beam in terms of the design variables and provides the basis of choice between alternative acceptable design, is taken as follows:

$$C(V) = C_c A_c + C_s A_s + C_f P_f \quad (1)$$

where $A_c$ and $A_s$ are the areas of concrete and prestressing steel, respectively. $C_c$, $C_s$ and $C_f$ are the cost of concrete including the cost of transportation, cost of steel and forming, respectively. $P_f$ is the perimeter of the cross-section. By minimizing this function an optimum design can be obtained. The limit between acceptable and unacceptable designs is governed by side and behaviour constraints. Side constraints are basically limits on the design variables. They are prescribed to satisfy CPCI code requirements, and/or to impose certain design conditions that the resulting cross-section be of a particular shape. The behaviour constraints are imposed to ensure the performance of the beam at service conditions at all load stages that may be critical during the life of the structure, as specified by the CPCI code, as well as to meet the strength requirements specified in the code.

The preceding constrained minimization problem is converted to a sequence of unconstrained minimization problem by using the penalty function method of Fiacco and McCormick (Ref. 1), in order to be able to use one of the powerful unconstrained minimization method. In this work, the Fletcher and Powell minimization method, (Ref. 2), which is considered the most powerful unconstrained minimization method, is used. The resulting function, F(V) can be expressed as follows:

$$F(V) = C(V) + P(V) \quad (2)$$

where the penalty function, P(V) is given by,

$$P(V) = R \sum_{j=1,2}^{q} 1/g_j(V) \quad (3)$$

and $g_j(V) \geq 0$ is the $j^{th}$ constraint function $j = 1,2,\ldots,q$ and q is the total number of side and behaviour constraints. Also, R is an arbitrary constant greater than zero which represents the relative weight of the constrained functions in the F(V) function.



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Proceedings of the Eighth Canadian Congress of Applied Mechanics, Moncton, June 7-12 1981.

The minimization procedure outlined in this work has been implemented using a FORTRAN IV program. Numerous numerical results indicate that the present structural synthesis capability is an efficient and convenient way for selecting minimum cost design for prefabricated prestressed concrete beams of various shapes and subjected to different loading conditions. Results also indicate that due to the presence of relative minima in the design space, it is not necessary to chose a good initial design. That is, a relatively costly initial design does not affect the minimum cost obtained. In fact, for such cases the optimum design obtained may have less cost than that resulting from an initially good design. It is pointed out that the major reduction in the cost for all the cases considered occured after only one trial. As a result, an optimum design can be achieved after a relatively short computer-run time.

In the following, sample results are presented for a simply supported beam which is desired to be of an I shape with a limited depth of less than 25 in., starting with an initial design of a rectangular shape. The initial and the optimum design obtained are shown in figures (2a, 2b), respectively. In table 1 the costs and operational informations are listed. It can be realized that a reduction of the cost from 24 $ / ft. to 7.8 $ / ft is achieved after only one trial, and that the other two computer runings have minor effect on the overall cost. It is noted that the optimum obtained is a constrained optimum, which is the case for all the other designs considered in this work.


REFERENCES

1. Fiacco, A.V. and McCormick, G.P., Computational Algorithm for the Sequential Un-Constrained Minimization Technique for Non-linear Programming, Management Science, Vol. 10, No. 2 (Jan. 1964), pp. 601-617.

2. Fletcher, R. and Powell, M.J.D., A rapidly Convergent Descent Method for Minimization, Computer Journal, Vol. 6, (1963), pp. 163-168.


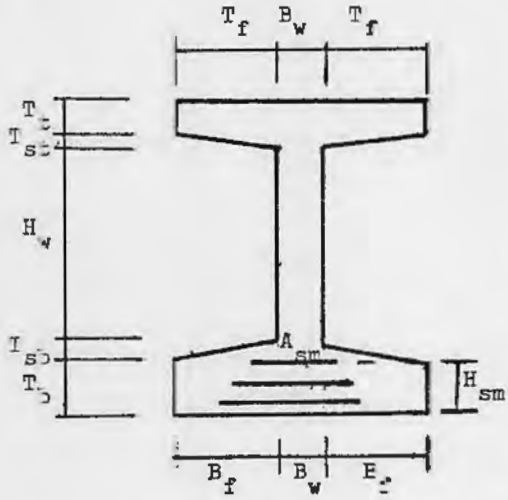


Fig. 1 Design Variables

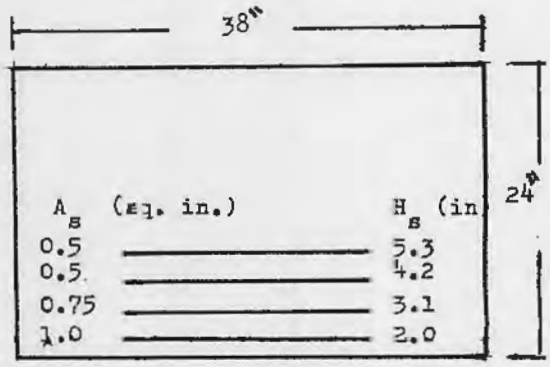


Fig. 2.a. Initial Design

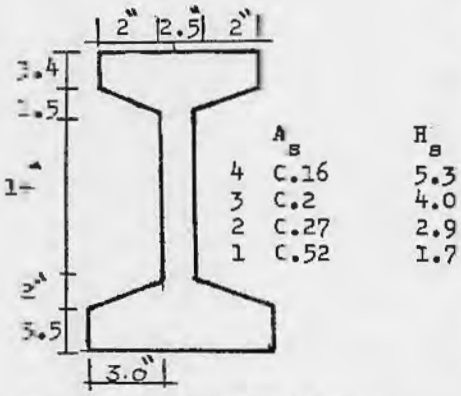


Fig. 2.b. Optimum Design

| Design | R | Shape | Cost $ /ft. | Funation Value F(V) |
|---|---|---|---|---|
| Initial | -- | □ | 24.0 | 35.5 |
| | 0.1 | I | 7.8 | 18.5 |
| | 0.01 | I | 6.65 | 7.99 |
| Optimum | 0.001 | I | 6.30 | 6.5 |

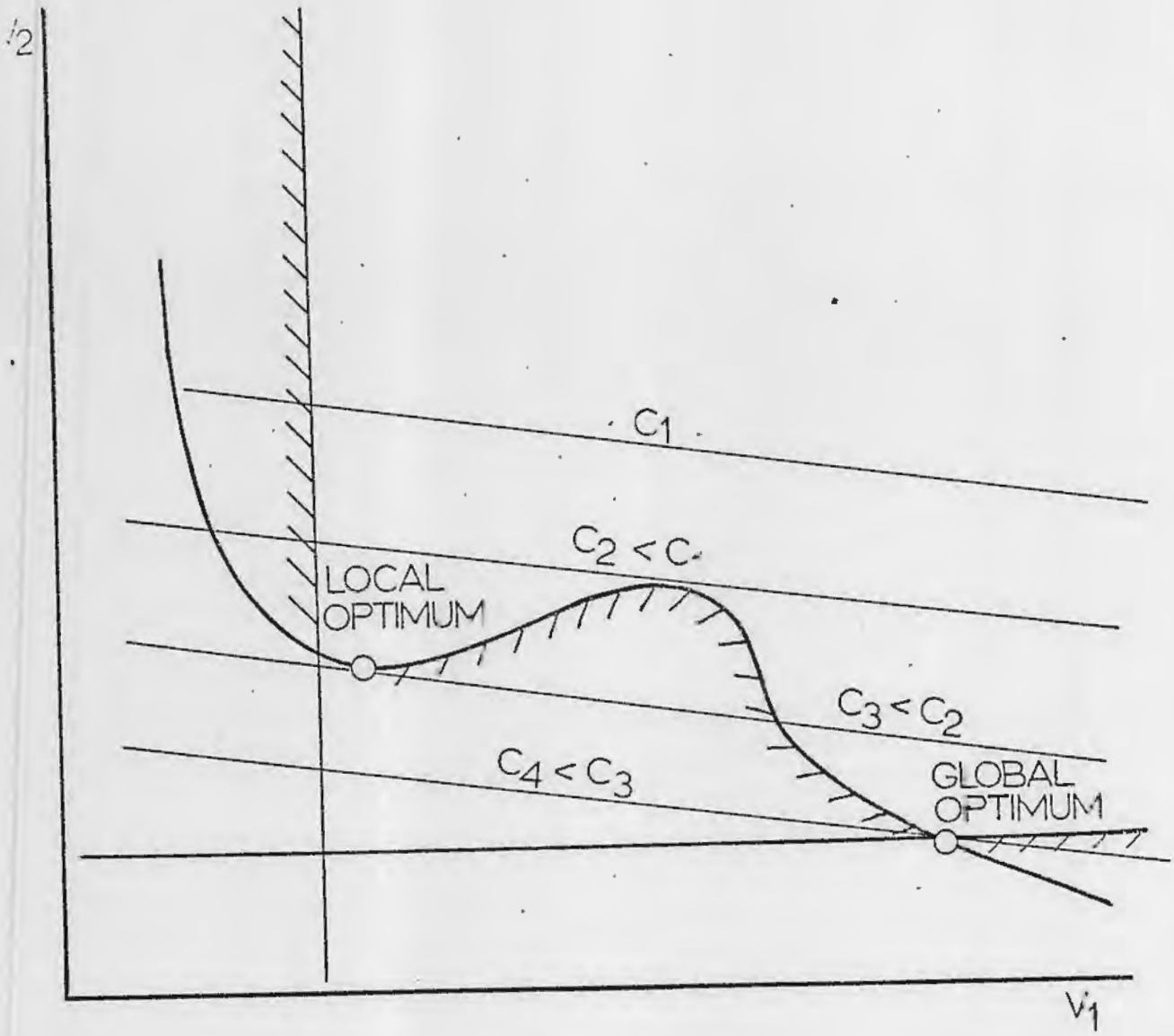


Fig. 3 RELATIVE MINIMA

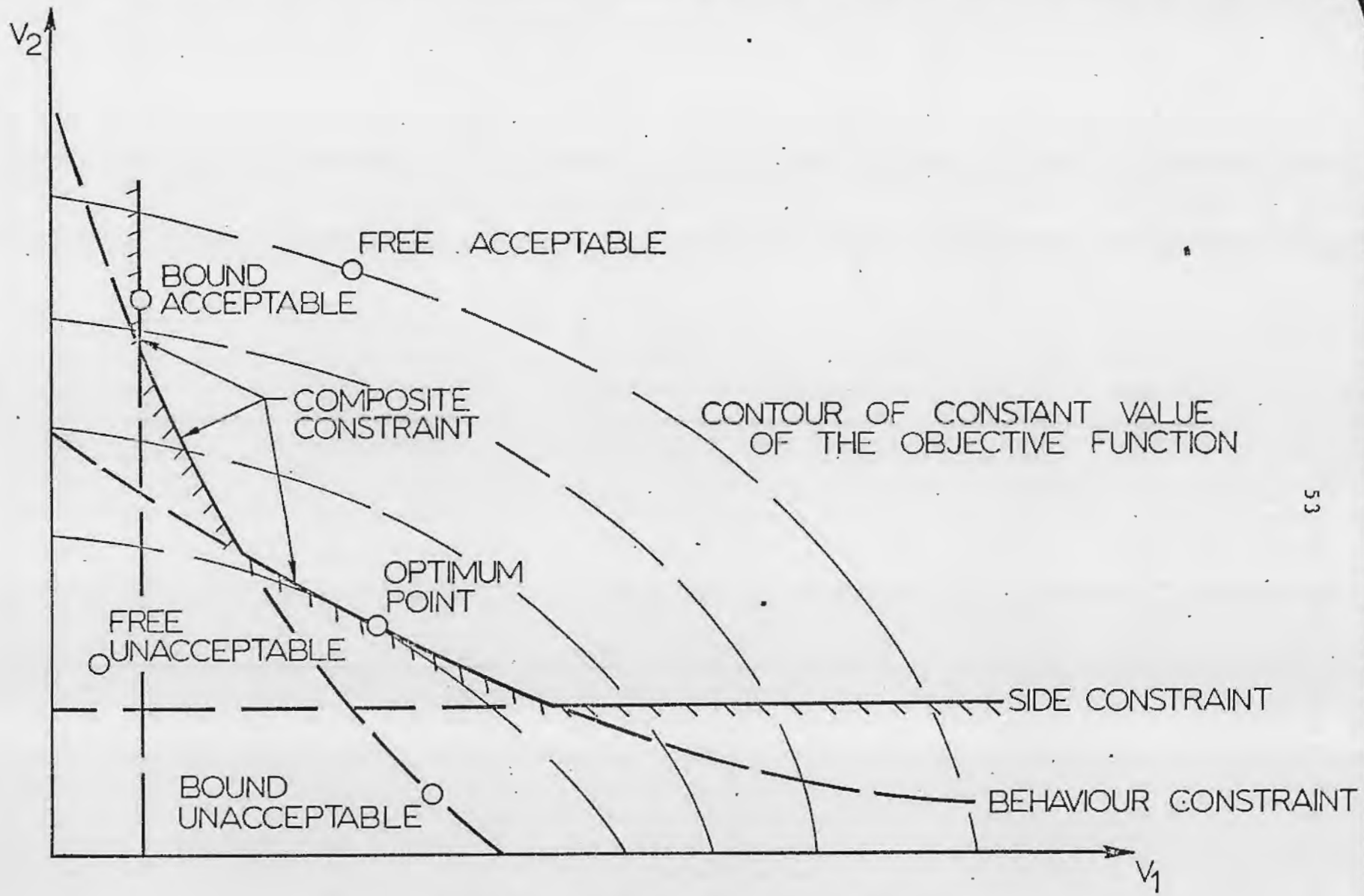


Fig. 1 DESIGN SPACE NOMENCLATURE