POWER STATION ECONOMICS

by: Dr. METWALY EL SHARKAWY

Faculty of Engineering,
Ain Shams University,
Cairo.,

by: Dr. METWALY EL-SHARKAWY

1- POWER STATION ECONOMICS

1.1. Economic Selection Of Plant:

Economic selection of plant is based on comparing the total annual costs of generating the required amount of electrical energy as per the specified conditions using different alternatives of station design. The total annual cost of generation can be augmented in the following two main items:

1- Annual fixed cost.

2- Annual operating cost.

For a given design, as more sophisticated equipment is added to improve station efficiency, the total investment increases and with it the annual fixed costs. As shown in Fig.1, the general relation between annual fixed costs and capital investment can be expressed as:

$$CF = RA$$

Where: CF= annual fixed cost, L.E.

R = fixed charge rate, as decimal.

A = capital investment ,L.E.

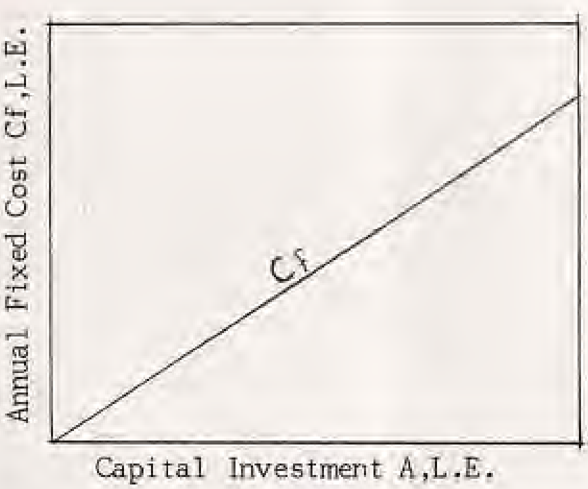


Fig.(1) Annual Fixed Cost

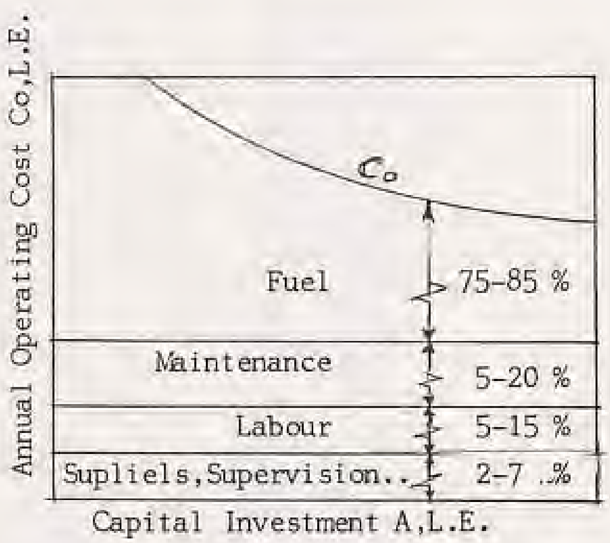


Fig.(2) Annual Operating Cost.

The benefit of the higher effeciency resulting from increased investment is realized in lower fuel and maintenance costs. Maintenance and repair are generally dependent on the amount of fuel burned. But labour,supervision and supplies costs can be considered constant in most cases.

The annual operating costs C include the costs of fuel,maintenance,labour .....etc, and it is graphically illustrated in Fig.2.

The total annual cost C is obtained by adding the total annual fixed and operating costs,i.e., $C = Cf + Co$

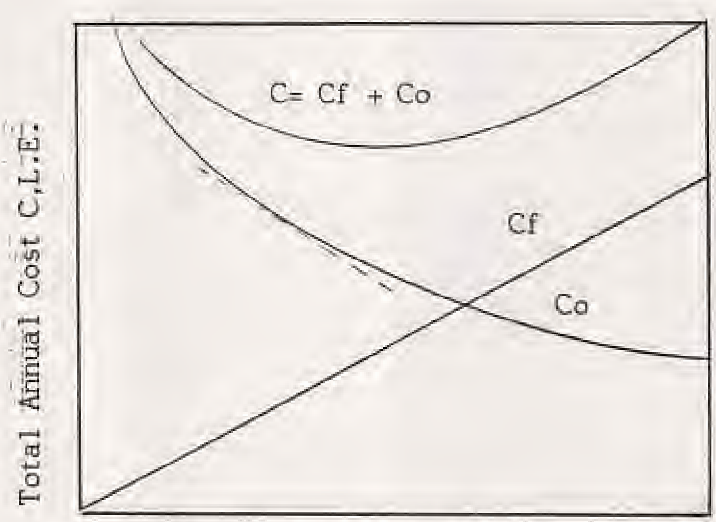
The relation between the total annual costs and the capital investment is graphically illustrated in Fig.3. It is clearly shown that there is some value of the capital investment A giving a minimum value of the total annual costs . This minimum takes place when $\frac{dc}{da} = 0$

Substituting $C = Cf + Co$, we get

$$\frac{dc}{dA} = \frac{dCf}{dA} + \frac{dCo}{dA} = 0$$

Which gives: $\frac{dCo}{dA} = - \frac{dCf}{dA} = -R$

This shows that the total annual costs will be minimum only when the slopes of the annual fixed and annual operating cost curves become numerically equal but opposite in sign.



Capital Investment A, L.E.
Fig.(3) Total Annual Cost

1.2. PLANT CAPACITY

After defining the load requirements in the form of a load duration curve, the capacity of the plant can be determined. The plant must be equal at least to the maximum demand of the load. But the station capacity is generally selected to give some reserve capacity. This reserve capacity differs from system to system according to size and number of units operated in the system and the security policy accepted by the management. In some of the large power-supply systems, made up of several generating units, design ruleshave been adopted to maintain a total installed capacity equal to the expected maximum demand plus the capacity of the two largest units. This is based on the expectation that at the time of the maximum demand one of the largest units or an equivalent amount of capacity may be out of service for over-hauling. Then, with all the other units actively operating, the peale load can still be carried if the other largest unit should fail owing to mechanical breakdown or operating error.

1.3. SIZE AND NUMBER OF UNITS IN A NEW STATION

The size and number of units to be installed in a new station is affected by many factors. One important factor is whether this new plant is being designed to supply a load in a completely newely dectiefied area,i.e.; will form its own system, or to supply power to an existing system. In the first case, the proposed capacity of the plant, which is taken equal to the estimated maximum demand plus the planned reserve capacity, may be installed in one unit of equipment if an interruption of service can be tolerated at any time. However, economical considerations may dictate in case of comparatively load factors to split the capacity among two or more units.

In most cases the interruption of service cannot be tolerated , so , if the load to be served is a small one it may pay to install two units of equipment each being capable of suppling the maximum demand independently.

In the second case, i,e; when the new plant is being designed to supply an existing system, the size and number of units to be installed in the new plant will be based on the following considerations:

1- The expected rate of increase of the maximum demand over a period of years.
2- The general design policy established by the system management.
3- The available space for units to be added.

If the expected rate of size of the maximum demand is increasing and shows promise of continuing into the future, the capacity of new units may exceed the capacity of any existing unit with the expectancy of saving on total costs over a period of years. Otherwise, the capacity of new units very likely will be selected equal to that of the largest existing unit.

1.4. PLANT CAPACITY BASED ON PROBABILITY ANALYSIS

At the present time the probability theory is being used by most of the electric power utilities to estimate their probable system outages and to arrive at a suitable reserve capacity based on specified security levels.

Considering a system having N units and assuming that the forced outage rates of these units, experssed as the ratio of numbers of days outage to the number of days outage to the number of days in the year, are : $Q1, Q2, \ldots, Qn$ respectively. The corresponding operating rates of the units $1, 2, \ldots, n$ will be $P1, P2, \ldots, Pn$ respectively, where $P1 + Q1 = 1$, $P2 + Q2 = 1, \ldots, Pn+Qn = 1$. Therefore : $(P1 + Q1)(P2 + Q2)\ldots\ldots(Pn + Qn) = 1$

This equation can be easily used to find the probable different combinations of units out of service and in service. For a system having N similar units, i.e; $P1 = P2 = \ldots = Pn = P$ and $Q1 = Q2 = \ldots = Qn = Q$,

The above equation reduced to:

$$(P + Q)^n = 1$$

To make a complete analysis, which may be done later, the outage probabilities are integrated with the system load - duration curve to measure the total loss of kilowatthour generation for different unit sizes before deciding which size is to be recommended for use.

(2) ECONOMIC OPERATION OF POWER STATIONS

2.1. STATION PERFORMANCE CURVES

Performance of generating plants is compared by their average thermal efficiencies over a period of time. The average thermal efficiency of a plant is defined as the ratio of useful energy output during the period to the total energy input for the period. The condition of copmarizon is that all plants should be operated at the same conditions,i.e.; using the same cooling-water temperature, the same shape of load-duration curve, the same total output and the same quality of fuel. If plants are operated at different conditions, which is the most probable case, their performance cannct be compared before correcting them to the same controlling conditions.

The performance of a generating unit is derived usually from test results for individual equipment and can be graphically represented by the relation between the output power, P, of the unit measured in KW or MW and the corresponding energy input, I , per hour operation measured in millions of kilocalories / hr. The shape of the input - output curve of a station may be as shown in Fig. 4a.

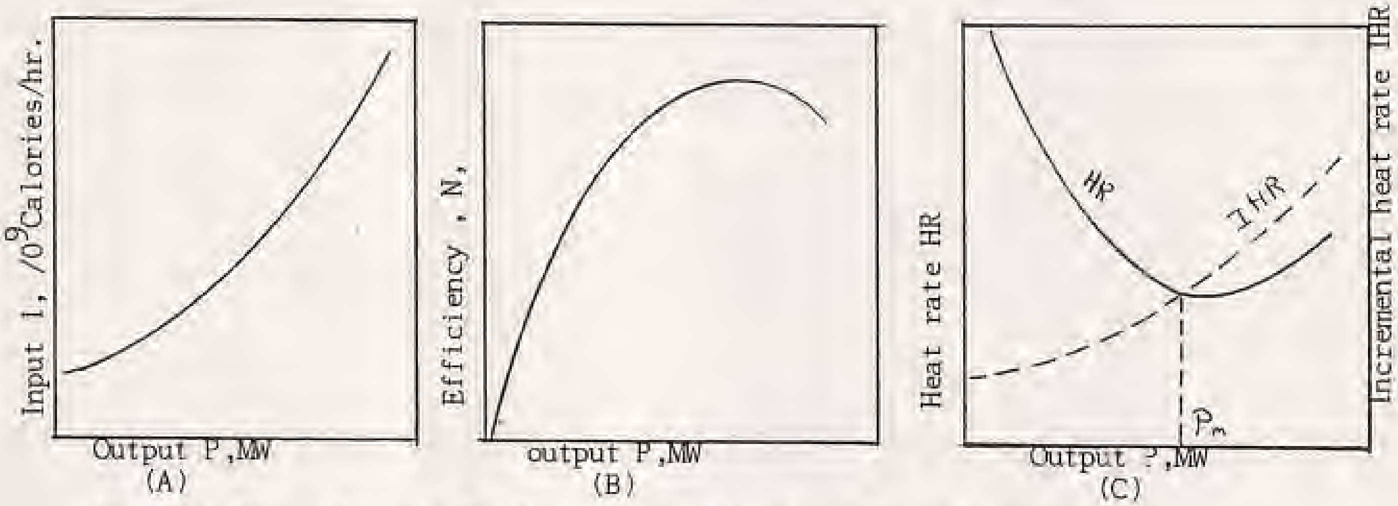


Fig.(4) Power Station Performances. (A) Input-Output Curve;
(B) Efficiency Curve ;
(C) Heat rate and Incremental heat rate Curves.

A more expressive curve for the unit performance is the efficiency curve shown in Fig. 4b. This curve can be obtained directly from the input-output curve using the relation:

$$N = \frac{0.8604\ P}{1} \times 100 \quad \text{percent}$$

Where P is the output power in MW and I is heat input measured in Gigacalories/Hr.

The form of the efficiency curve is usually as shown in Fig.4.b. In addition to the efficiency curve, the heat rate HR and incremental heat rate IHR curves shown in Fig. 4.C. are also used as unit performance curves. The heat rate curve is derived by taking at each load the corresponding input, then

$$HR = \frac{I}{P} = \frac{86.04}{N\%} \quad \text{G.Calories/MWH}$$

is plotted against the corresponding value of P.
If the input - output curve is expressed also mathematically in the form of a polynomial as follows:

$$I = a + bP + C\,P^2 + o/P^3$$

Then the heat-rate will be given by:

$$HR = \frac{a}{P} + b + cP + dP^2$$

The incremental heat-rate curve is derived from the input-output curve by finding at any load P the additional input dI required for a given additionel output dP; i.e.; the incremental heat rate (IHR) defined as:

$$IHR = \frac{dI}{dP} \quad \text{G.Calories}$$

Is plotted versus the output power P for the full range of the input-output curve. Mathematically the IHR is the slope of the I-P curve, and physically is the amount of additional input required to produce an added unit of output at any given load. The HR - P and IHR -P curves are shown in Fig. 4.C.

Now a question arises about the loading of the generating unit at which it has the highest efficiency. Looking at the efficiency curve, it is easy to find that the unit has its maximum efficiency when $\frac{dN}{dI} = 0$, or

$$\frac{d}{dI}\frac{(86.04\,P)}{I} = 86.04\ \frac{d\,(\,P/I\,)}{dI}$$

$$= 86.04\left(\frac{1}{I}\frac{dP}{dI} - \frac{P}{I^2}\right) = 0$$

which gives : $\frac{P}{I} = \frac{dP}{dI}$ or $\frac{I}{P} = \frac{dI}{dP}$

This shows that the unit has the highst efficiency only when its heat-rate (HR) equals its inremental heat -rate (IHR).

Referring to Fig. 4C. it is easy to show that the unit will have maximum efficiency if loaded by the power Pm corresponding to the point of intersection of the HR and IHR curves.

2.2. AVERAGE HEAT RATE

If the generating unit is operated to supply a load defined by a load - duration curve as shown in Fig. 5A, the input to the unit varies according to the output, which means that it will operate at different heat rates. The average heat-rate of the unit for the whole time period of the load duration curve is defined as:

$$\text{Average HR} = \frac{\text{total input during the period}}{\text{total output during the period}}$$

To find the average HR according to this definition, we have to make use of both the load - duration curve (Fig.5A) and the input-output curve of the unit (Fig. 5B).

The total output of the unit during the whole period is given by :

$$\sum P\,\Delta t = P_1\Delta t_1 + P_2\Delta t_2 + \ldots\ldots\ldots + P_6\Delta t_6$$

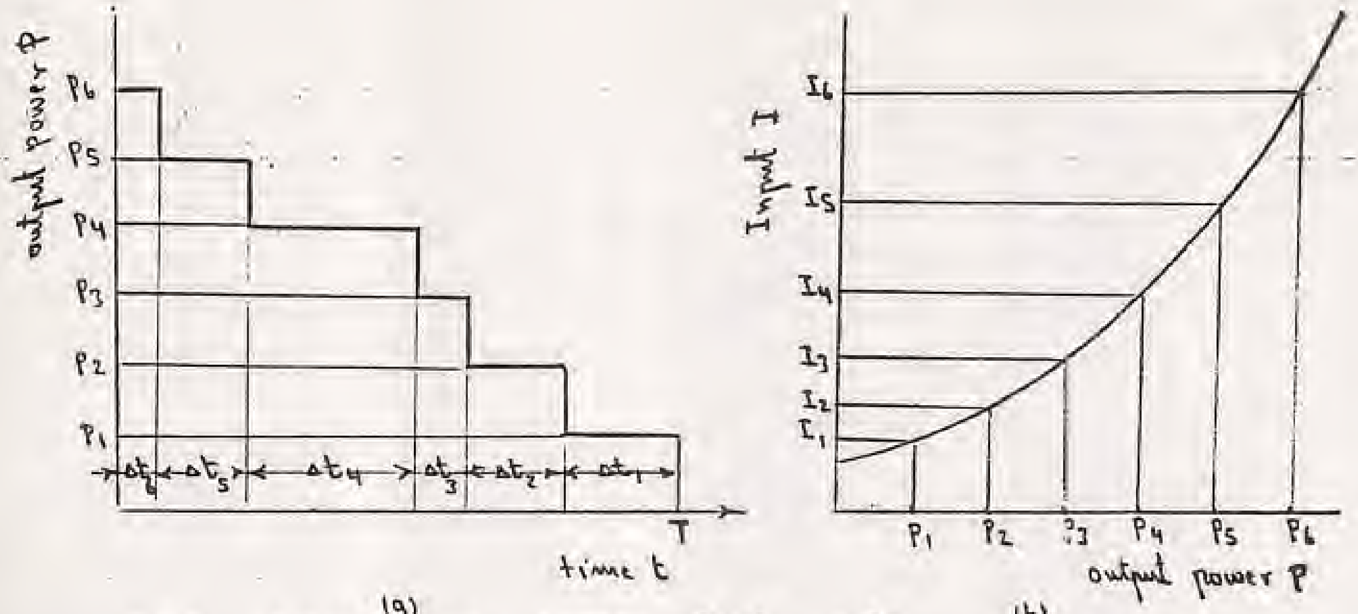


Fig.5 Determining average heat rate.

The total output of the unit will be measure in MWH if P is in MW and T in hours. Also after finding the unit input $I_1, I_2, \ldots\ldots I_6$ corresponding to the outputs $P_1, P_2, \ldots\ldots P_6$ respectively from Fig.5B, we can find the total unit input during the whole period as:

$$\Sigma I \Delta t = I_1 \Delta t_1 + I_2 \Delta t_2 + \ldots\ldots I_6 \Delta t_6 . \text{ G.Calies.}$$

The average heat rate will be then given by:

$$\text{Average HR} = \frac{I_1 \Delta t_1 + I_2 \Delta t_2 + \ldots\ldots + I_6 \Delta t_6}{P_1 \Delta t_1 + P_2 \Delta t_2 + \ldots\ldots + P_6 \Delta t_6} = \frac{\text{Iav}}{\text{Pav}}$$

Where:

Iav is the average input to the unit = $\frac{\Sigma I \Delta t}{T}$

And: Pav is the average output of the unit = $\frac{\Sigma P \Delta t}{T}$

T is the total hours of the period = $\Sigma \Delta t = \Delta t_1 + \Delta t_2 + \ldots\ldots + \Delta t_6$

Example: The input -output of a 20 MW generating station is defined by:

$$I = 7.56 + 0.126P + 0.164 P_2$$

Where I is in G.Cal. per hour and P is in MW. Find the average heat rate of this station for a day when it is operating at a load of 20 MW for 12 hr. and kept hot a zero load for the remaining 12 hours. Compare this average heat rate with the heat rate that would obtain if the same energy were produced for the day at a constant 24 hrs., i.e. at 100% load factor.

Solution:

At: P= 0: I= 7.56 + 0.126 x 0 + 0.164 = 7.56 G.Cal/Hr.

At: P=20 MW: I= 7.56+ 0.126 x 0.164 x 20^2 = 75.68 G.Cal./Hr.

P t= 0 x 12 + 20x 12 = 240 MW hr.

I t= 7.56 x 12 + 75.68 x 12 = 998.88 G.Cal.

Average daily heat rate HR: $\frac{\Sigma I \Delta t}{\Sigma P \Delta t} = \frac{998.88}{240}$ = 4.162 G.Cal/MWh.

Average load: $\frac{240}{24}$ = 10 MW

At P = 10 MW for 24 hr. I= 7.56+0.126 x 10 +0.164 x100= 25.22 G.Cal/hr.

$\Sigma I \Delta t$= 25.22 x 24 = 605.28 G.Cal.

Corresponding HR = $\frac{605.28}{240}$ = 2.522 G.Cal.

This shows that, if it were possible to redistribute the generation of the total daily output to a constant rate for the 24 hrs., a saving of 4.162 -2.522= 1.640 G.Cal./ MWhr. could be achieved.

2.3. ECONOMIC LOAD SHARING BETWEEN GENERATING UNITS IN A STATION

It has been shown in the previous section that generation economics are affected not only by station characteristics but also by the operating load curve. The study was limited in the previous section to a single-unit station. A system having more than one generating unit has the proper load divisionas a problem. Improper load division may appreciably decrease the thermal effeciency of the station as a whole, and consequently the cost of generation per MWhr. will be increased.

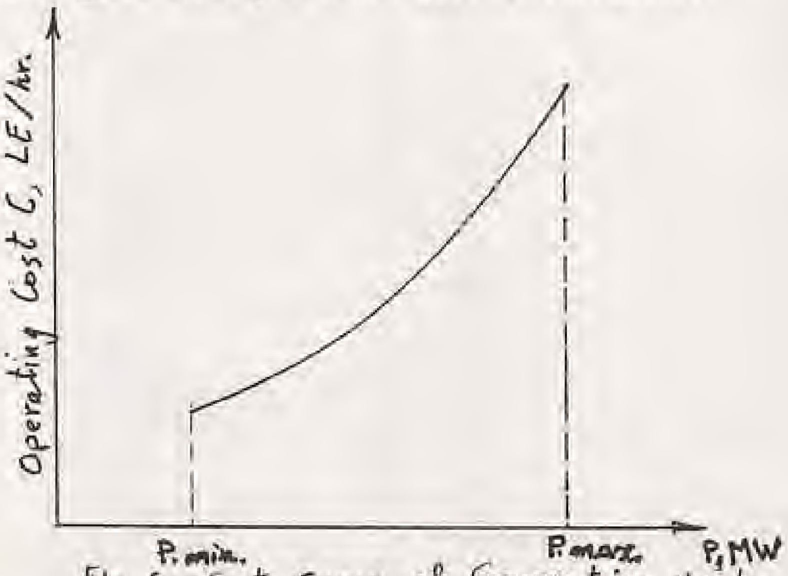


Fig. 6. Cost Curve of Generating Units

2.3.1. THE COST CURVE OF A GENERATING UNIT

The study in this section ,as it was in previous sections,is concentrated on fuel fired stations. This means that a generating unit consists of a boiler,turbine and generator. The major compound of a generator operating cost, as has been previosly shown, is the cost of fuel consumption per hour. So it is easy to convert the input-output curve of generating units given in Fig.4A into a cost curve by multipling the input quantities (G.Cal/Hr.) by the price of fuel (L.E./G.Cal.) to get the fuel cosst (L.E./hr.). This cist can be taken as approximately equal to the operating cost of the unit (L.E./hr.). A typical cost curve is shown in Fig.6 where P min. is the minimum loading limit below which it is uneconomical or technically unfeasible to operate the unit and P max. is the maximum output limit. It is important to note that the actual input-output and cost curves of generating units has discontinuities at steam valve openings which are not shown in figures given here.

The cost curve of unit I in a station having N units can be expressed as:

Ci = Ci (Pi) L.E./Hr. at output power Pi MW.

This function can be approximated , by applying a suitable curve fitting method, to a second degree polynomial with a sufficient degree of accuracy as follows:

$$C_i = \tfrac{1}{2}a_i\, P_i^{\,2} + b_i\, P_i + d_i \qquad \text{L.E./hr.}$$

Where ai, bi and di are constants for the unit I.

The slope of the cost curve at any output power P_i,i.e., $\frac{dc_i}{dp_i}$ is called the incremantal fuel cost (IC)i and is expressed in L.E./MWhr.

If the cost curve is approximated to a second order polynominal,the incremental cost will be given by: $(IC)_i = a_i P_i + b_i$ L.E./MWhr.

Which is a linear relationship. Alternatively, we can fit a polynominal of suitable degree to represent the IC -P curve in the inverse form:

$$P_i = \alpha_i\, B\, (IC)_i + \alpha_i\, (IC)^2_i + \ldots\ldots.$$

2.3.2. ECONOMICAL LOAD SHARING BY TWO UNITS IN A STATION

Considering firestly a two-unit station that is asked to supply a load connected to its fuses by a power PD at the lowest possible cost. It is required to find the share of each unit in the generated power to fulfil this requirement. Let the share of unit 1 be P_1 and that of unit 2 is P_2. Assuming the corresponding cost of generation to C_1 and C_2 for units 1 and 2 respectively.

The total cost of generation will be:

$$C = C_1 + C_2 = C_1(P_1) + C_2(P_2).$$

Also P_1 and P_2 are related by:

$$P_1 + P_2 = P_d$$

The cost of generation é will be minimum only if $\frac{dc}{dP_1} = 0$

Which gives $$\frac{dc_1}{dP_1} = -\frac{dc_2}{dP_1} = -\frac{dc_2}{dP_2}\frac{dP_2}{dP_1}$$

But the equation $P_1 + P_2 = P_d$ gives $\frac{dP_2}{dP_1} = -1$

which when substituted in the last equation yields: $\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2}$

Or: $(IC)_1 = (IC)_2$

This means that the condition for minimum lost of generation in a two-unit station is to operate the two units at equal incremental costs.

<u>Example:</u> The incremental fuel costs in L.E./MWhr for a plant consisting of two units are:

$$(IC)_1 = \frac{dC_1}{dP_1} = 0.027\ P_1 + 5.32 \qquad \text{LE/MWH.}$$

$$(IC)_2 = \frac{dC_2}{dP_2} = 0.033\ P_2 + 4.00 \qquad \text{LE/MWH.}$$

Assume , that both units are operating all the time to supply a load having the following load-duration curve:

Load, MW	40	80	120	160	200	250
Duration, t	6	6	4	4	2	2

and that the maximum and minimum loads on each unit are to 125 and 20 MW respectively.

a) Construct the daily loading table for both units if they are operated to supply the given load at minimum possible cost.

b) Calculate the daily saving due to econominal rather than equal load sharing between the two units.

- Solution:

a) Economical load sharing based on equal incremental fuel costs can be calculated from the following equation:

$$P_1 = \frac{0.033\ PD - 1.32}{0.06}$$

Where PD is the station loading.

The following table gives the required economical and constrained loading of both units:

PD, MW	40	80	120	160	200	250
t, hr.	6	6	4	4	2	2
Optimum loading P_1	0	22	44	66	88	115 5
Of units, MW P_2	40	58	76	94	112	134 5
Constrained opt. P_1	20	22	44	66	88	125
Min. loading, MW P_2	20	58	76	94	112	125

b) The optimum loading tabe obtained shows that for all loads the unit 2 is asked to carry more than half the load, which means that its operating cost for equally divided loads will be less than for optimum load sharing.

Total daily increase in cost of operating unit 2 at tabulated loads rather than equal load shring is given by:

$$6\times\left[\int_{20}^{2}(0.033\,P_2+4.0)\,dp_2+\int_{40}^{58}(0.033\,P_2+4.0)\,dp_2\right]+$$

$$4\times\left[\int_{40}^{76}(0.033)P_2+4.0)\,dp_2+\int_{80}^{94}(0.033\,P_2+4.0)\,dp_2\right]+$$

$$2\times\left[\int_{100}^{112}(0.033\,P_2+4.0)\,dP_2+\int_{125}^{115}(0.033\,P_2+4.0)\,dp_2\right]=$$

$$=\frac{0.033}{2}\left[6\times(58^2-40^2)+4\times(76^2-60^2+94^2-80^2)+2\times(112^2-100^2)\right]$$

$$+\ 4\times\left[6\times(58-40)+4\times(76-60+94-80)+2\times(112-100)\right]$$

$$=\ \underline{\underline{1570.98\ \text{L.E}}}$$

Total daily decrease in cost of operating unit 1 at tabulated loads rather than equal load sharing is given by:

$$6\times\left[\int_{20}^{20}(0.027\,P_1+5.32)+dP_1+\int_{22}^{40}(0.027\,P_1+5.32)\,dP_1\right]+$$

$$+\ 4\times\left[\int_{44}^{60}(0.027\,P_1+5.32)\,dP_1+\int_{66}^{80}(0.027\,P_1+5.32)\,dP_1\right]+$$

$$+\ 2\times\left[\int_{88}^{100}(0.027\,P_1+5.32)\,dP_1+\int_{125}^{125}(0.027\,P_1+5.32)\,dP_1\right]=$$

$$=\ \underline{\underline{1692.18\ \text{L.E.}}}$$

Total daily saving = 1692.18 - 1570.98 = 121.2 L.E.

Corresponding annual saving = 44268.3 L.E.

2.3.3. Optimum Load sharing for Multi-Unit Stations:

The simple procedure followed in deriving the necessary conditions for optimum load sharing in a two-unit station cannot be easily followed for getting the optimality conditions for load sharing in multi-unit stations. So it is better to have a more general formulation for the multi-unit case. Assuming that N units are selected to supply the load PD during the period under study and that these units are selected according to acceptable rules of scheduling units in the station. Obviously the following constraints shall be considered while supplying the combined load of the station.

$$\sum_{i=1}^{m} P_{i\,max} \geq PD \quad (1)$$

$$P_{i\,min} \leq P_i \leq P_{i\,max}, \quad i = 1, 2, \ldots, m \quad (2)$$

In addition to that the considerations of spinning reserve, to be discussed later, requires that

$$\sum_{i=1}^{m} P_{i\,max} > PD \quad (3)$$

By a proper margin, which means that Eq.(3) is a strict inequality.

Since the effect of reactive loading of generators on their active power losses in of negligible order, it is assumed here that the manner in which the reactive load of the station is shared among various on-time generators has negligible effect on the economy of generation.

The problem now is " what is the optimal manner in which the load demand PD must be shared by the generators on the fuse?" The solution to this problem is to minimize the cost function.

$$C = \sum_{i=1}^{m} C_i(P_i) \quad (4)$$

Under the equality constraint of meeting the load demand, i.e.

$$\sum_{i=1}^{m} P_i - PD = 0 \quad (5)$$

and the inequality constraints given by Eqs. (1),(2) and (3).
This problem is a typical separble non-linear programming problem. Assuming that the inequality constraints are not effective at present, the problem can be solved by the method of Lagrange Multipliers. The Lagranian is defined as:

$$\mathcal{L} = \sum_{i=1}^{m} C_i(P_i) - \lambda \left[\sum_{i=1}^{m} P_i - PD\right] \quad (6)$$

Where λ is the Lagrange multplier.

The condition of Optimality is

$$\frac{\partial d}{\partial pi} = 0 \quad \text{or} \quad \frac{dc^i}{dp^i} = \lambda \quad \text{for } i = 1, 2, \ldots, n \qquad (7)$$

Where $\frac{dci}{dPi}$ is the incremental cost of the i th generator (LE/MWh).

Equation (7) can be rewritten as:

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \cdots = \frac{dC_n}{dP_n} = \lambda$$

Which means that the optimal loading of generator corresponds to the equal incremental cost point of all generators. The numerical solution of the optimal load sharing problem can be easily done applying iterative techniques using digital computers. The procedure of solution can be arranged as follows:

1. Choose a trial value of ,i.e. IC = (IC) .
2. Solve equations (8) for Pi, i= 1,2, ...,n.
3. $F \ \left| \sum_{i=1} P_i - P_D \right| < E$

(a specified acceptable accuracy of solution) , the optimal solution is reached. Otherwise,

4. Increase (IC) by an increment Δ (IC) if $\left[\sum_{i=1}^{n} P_i - P_D \right] < 0$, or decrease (IC) by the increment Δ (IC) if $\left[\sum_{i=1} P_i - P_D \right] > 0$ and repeat from step 2.
5. If during the iterative process the loading Pj of any generator J reaches the limit Pj min. or Pj max. ,the share of this generator in the demand should be held fixed at the reached limit and the remaining load power $[P_D - (P_j \text{ max. or } P_j \text{ min.}$]is then shared between the remaining generators on equal incremental cost basis.

2.3.4. OPTIMUM SCHEDULING OF GENERATING UNITS IN A STATION

Scheduling of generating stations units means determining the units of the station that should be operated to supply a particular load. This problem is also called the unit commitment (UC) problem. It is initiated by economical considerations, since it is clear that it is not economical to run all the units available in a station all the time whatever may be the value of the load on it. It is better to clarify this point through an example.

Example: The two units considered in the previous example are operated to supply a load having a daily load curve as follows:

Load , MW	76	220	76
Time of day:	From 12 m.n. to 6 a.m.	From 6 a.m. to 6 p.m.	from 6 p.m. to 12 m.n.

The cost curves of the two units are given by:

$$C_1 = 0.0130\,P_1^2 + 5.32\,P_1 + 16.0 \quad L.E./hr., \text{ and}$$
$$C_2 = 0.0165\,P_2^2 + 4.00\,P_2 + 13.3 \quad L.E./hr.$$

The cost of taking either unit off the line and returning it to service is LE.25. Find if it is economical to keep both units in service continuesouly or to remove one of the units for the 12 hours of light load.

Solution:

Optimum load sharing on the basis of equal incremental for both units is as follows:

P ,MW	76	220
Economical P_1	19.8	99
Sharing,MW P_2	56.2	121
Constrained P_1	20	99
Sharing,MW P_2	56	121

Total cost of generation to supply the 220MW load for a 12 hr.period is:

$$C = C_1 + C_2$$
$$= \left[(0.0130 \times 99^2 + 5.32 \times 99 + 16.0) + (0.0165 \times 121^2 + 4.00 \times 121 + 13.3)\right] \times 12$$
$$= L.E.\ 6907.634$$

Total cost of generation to supply the 76 MW load for the other 12 hr.period is:

$$C = C_1 + C_2$$
$$= \left[(0.0130 \times 20^2 + 5.32 \times 20 + 16.0) + (0.0165 \times 56^2 + 4.00 \times 56 + 13.3)\right] \times 12$$
$$= L.E\ \ 4399.728$$

Total cost when both units are operating throughout the 24 hrs. period is:
L.E. 21907.362

By inspecting the two cost equations it is easy to find that unit 2 is more economical for light loads than unit 1, so if it is required to put one of the units off during the light load period, it will be economical to put unit 1 off. The total cost of generation during the light load period will be:

$$(0.0165 \times 76^2 + 4.00 \times 76 + 13.3) \times 12 = \text{L.E. } 4951.248$$

Total operating cost for the light load period will be the cost of generation plus the start-up cost of unit 1 , i.e.

4951.248 + 45.0 = LE 4996.248

comparing this with the earlier case, it is clear that it is more economical to put unit 1 off during the light level period and to start it up again to take part in the larger load than keeping it running all the time.

It is easy to see that if the start-up cost is. LE. 50, than it will be more economical to keep both units running all the time.

This example shows that the unit commitment problem is of economical importance. A simple but sub-optional approach to this problem is to impose prority ordering based on unit efficiency; i.e. to commit the most efficient unit firstly and then the less efficient one and so on as the load increases.

A simple, but highly time consuming, way of finding the most economical combination of units to supply a given load, is to try all possible combinations of units in the station that can supply this load, to divide the load among the units of each combination using the equations (IC) i = for i = 1,2, ..., m where m is the number of generators selected to operate in parallel in the combination. The object is to determine the combination which has the least operating cost.

OPTIMUM UNIT COMMITMENT APPLYING DYNAMIC PROGRAMMING.

The application of dynamic programming methods to find a unit commitment (UC) table for a complete load cycle can lead to a considerable saving in computation effort and time as compared to the above method. The reason for this saving is that the unit combinations to be tried applying dynamic programming methods are much reduced in number; in addition it is not necessary to solve the coordination equations for optimum load sharing.

In the dynamic programming approach the total number of units, N, available in the station, their individual cost curves and the load curve on which the station will be operated are assumed to be all known in advance. Also, it will be assumed that the load on each unit or combination of units is changed in sufficiently small and iniform steps of Δp mw (e.g. 1 MW)

Starting arbitravily with any two units, the most economical combination is determined for all the discrete load levels of the combined output of the two units. At each load level the most economic answer may be to run either unit or both units with a certain load sharing between the two. The most economical cost curve obtained in the discrete form for the combination of the two units can be viewed as the cost curve of a single equivalent unit. A third unit is now added to the single unit equivalent to the first two and the same procedure is repeated to find the cost ccurve of a new single unit equivalent to the three combined units. It is important to note here that in this procedure there is no need to make combinations of the first and third units or of the second and third units since all of these combinations are already considered in the second dynamic programming step. The process is repeated till all available units are included in the study. This approach has the advantage that it is quite easy to determine the optimum manner of loading (k+1) units if the optimal way of loading k units is already known or determined.

Let a cost function $F_K(X)$ be defined as follows: -

$F_K(x)$ = The minimum cost in LE/HR of generating x MW by k units.
$f_K(y)$ = Cost of generating y MW by the Kth unit.
$F_{K-1}(x-y)$ = The minimum cost of generating (x-y) MW by the remaining (k-1) units.

According to the dynamic programming approach the following recursive relation.

$$F_K(x) = \text{minimum of } \{ f_K(y) + F_{K-1}(x-y) \} \quad \text{for all possible values of } y \qquad (9)$$

can be easily used to determine the combination of units yielding minimum operating cost for loads ranging in convenient steps, from the minimum permissible load of the smallest unit to the sum of the capacities of all available units. In this process the total minimum operating cost and the load shared by each unit of the optimal combination are automatically determined for each load level.

EXAMPLE:- A power station having four thermal generating units with parameters listed in the following table is required to supply a load of 9 MW. Determine the most economical UC tabulation.

UNIT	PMIN	PMAR	COST CURVE PARAMETERS (d=o) a(LE/MW^2)	b (LE/MW)
1	1.C	12.0	0.10	3.13
2	1.C	12.0	0.21	3.53
3	1.C	12.0	0.27	4.00
4	1.C	12.0	0.33	4.27

solution. let the load changes be in steps of 1 MW.

$$F_1(9) = F_1(9) = \tfrac{1}{2} a_1 P_1^2 + b_1 P_1 = 0.05 \times 9^2 + 3.13 \times 9 = LE\ 32.22/hr.$$

$$F_2(9) = \text{min. of } \{[F_2(0) + F_1(9)], [F_2(1) + F_1(8)], [F_2(2) + F_1(7)], \ldots, [F_2(9) + F_1(0)]\}$$

computing these quantities term-by-term and comparing, we get

$$F_2(9) = [F_2(2) + F_1(7)] = L.E.\ 31.84/hr.$$

In the same way, we can calculate $F_2(8), F_2(7), \ldots, F_2(1), F_2(0)$.

Using the recursive relation (9), we can now calculate F3(0), F3(1), ...F3(9)

$$F_3(9) = \text{min. of } \{[F_3(0) + F_2(9)], [F_3(1) + F_2(8)], \ldots, [F_3(9) + F_2(0)]\}$$
$$= [F_3(0) + F_2(9)] = L.E.\ 31.84/hr.$$

Proceeding similarly, we get

$$F_4(9) = [F_4(0) + F_3(9)] = L.E.\ 31.84/hr.$$

The obtained values for $F_1(9)$, $F_2(9)$, $F_3(9)$ and $F_4(9)$ lend to or conclusion that optimum units to be committed for a 9 MW load are 1 and 2 sharing the load as 7 MW and 2 MW respectively with a minimum operating cost of LE.31.84/HR.

It should be noted here that the solution accuraly is dependent on the step size. If a higher accuracy is required, the step size should be reduced. But care must be taken because reduced step size may result in a considerable increase in computation time and required storage capacity.

The described procedure is repeated for combined loads ranging from 1 MW to 48 MW (maximum power that can be delivered by the four units) in steps of 1 MW to prepare the optimum UC table shown below. It is easy to note that load values for which the unit commitment does not change are combined in one range.

The UC table is usually prepared only once for a given set of units, and as the load cycle on the station changes. It would only mean changes in starting and stopping of units with the basic unit commitment table remaining unchanged.

Unit commitment table for the studied systems.

load range MW	Units to be committed	Spinning capacity MW
1.5	1	12
6.13	1,2	24
14-18	1,2,3	36
19-48	1,2,3,4	48

2.3.5 Station Cost Curve

Using the UC table and increasing load in steps, the most economical operating cost of the station is calcultated for the complete range of stations capacity by applying the criteria of optimum load sharring among comitted units for each load value. The result is the overall station cost characteristic in the form of a set of data points. This set of data points can be used to apply a suitable curve fitting technique to obtain the required cost curve of the station in the form of a second or higher order polynomical. This curve can be used for economic load sharring among generating stations in an interconnected system.

2.3.5. SECURITY CONSTRAINED OPTIMAL UNIT COMMITMENT

Every electric utility is normally under obligation to provide its consumers a certain degree of continuity and quality of service. Therefore, economy and reliability must be properly coordinated in arriving at the operational unit commitment decision. A purely economic UC decision must be modified to take reliability requirments into consideration. As has been already explained, the only constraint taken into account while preparing the economic UC table was the fact that the total capacity on line should be at least equal to the load. The margin, if any, between the capacity of committed units and load was incidental. Under these conditions if one or more of the running units were subjected to a forced outage (random outage), it may not be possible to meet the load requirments. To bring a cold spare unit on steam and to synchronize it to take up the load will be taking several hours (2-3 hours), so that the load cannot be met for intolerably long periods of time. Therefore, to meet contingencies, the capacity of committed units must have a definite margin over the load requirements at all times. This margin is known as the spinning reserve and should ensure continuity of supply up to a certain extent of probable loss of generation capacity.

Since the unit which is to provide the spinning reserve at a particular time has to started several hours ahead, the problem of supply reliability (or security) has to be treated in totality over a period of one day. The analysis here depends on failure and repai rates of running units. The fail rate is the number of random failures of the unit per year, and the repair rate is the number of repairs per year. The failure and repair rates can be found from the past data of units(or other similar units elsewhere) nd are calculated as the inverse of mean time to failure (mean "up" time) and the mean time to repair (mean"down" time) respectively. Denoting the failure rate by and the repair rate by , we can write the probabilities (p or q) of a unit being in "up" (service) or "down"(forced outage) states at any time as

$$p = \frac{\mu}{\mu+\lambda} \tag{10}$$

$$q = \frac{\lambda}{\mu+\lambda} \tag{11}$$

The probabilities P and Q are also termed as availability and unavailability respectively.

For a system having N operating (running) units,the probability for the system to be in the state I defined by K units in service and (n-k) in forced outage is given by

$$P_i = \prod_{j=1}^{k} P_j \prod_{j=k+1}^{m} q_j \tag{12}$$

The probability that the available generation capacity (sum of capacities of units commited) at a particular hour is less than the system load at that time, is defined as:

$$S = \sum P_i \times r_i \tag{13}$$

Where Pi is the probability of system being in state i as defined by equation(12).
Ri is the probability that system state i causes breach of system security.

This formula is known as Patton's security function when system load is deterministic (i.e. known with xomplete certainty), Ri=1 if available capacity is less than load and O otherwise. In this sense is a quantitative estimate of system insecurity.
Theoretically equation (13) must be summed over all possible system states,what is very large, but since the probability of accurance of states with more than two units on forced outage at a time is very low,the summation is carried out practically over states reflecting a relatively small number of units on forced outage.
The security level of the system should not exceed a certain maximum tolerable insecurity level (MTIL), which is a figure to be determined by system management based on past experiency. Therefore,once the units to be commited at a particular load level are known from purely economic considerations,the security function is computed as per equation (13). If the value of exceeds MTIL,the economic UC schedule is modified by bringing-in the next most economical unit as per the UC table, is then recalculated and checked and the process is continued till $S \leqslant MTIL$. Practical experience shows that as the economic UC table has some inherent spinning reserve,rarely more than one iteration is found to be necessary. After getting a secure and economically optimal UC table for all individual periods of the load curve,such a table is to be checked to find if certain units have to be started and stopped more than once. If so, start-up cost of these units must be taken into consideration from the point of view of overall economy. This means that we have to examine whether or not it will be more economical to avoid restarting by continuing to run these units.

Example: Consider that the station for which the economical UC table had been obtained in the previous example is used to supply a load having the following daily load cycle:

Day time	12 m.n. to 4 a.m.	4-8	8-12 n	12n - 4 p.m.	4-8	8-12 m.n.
Load, MW	5	10	15	10	20	15

Construct the economical UC table for this load and check if it is secure in every period assuming identical failure and repair rates of 1 and 99 per year respectively for all the four units and that the system MTIL is 0.005.

Solution: The UC table shon below is directly obtained for the given load cycle using the previously prepared UC table in the last example.

Day time:	12 m.n. to 4 a.m.	4-8	8-12 n	12n - 4 p.m.	4-8	8-12 m.n.
Load, MW	5	10	15	10	20	15
Commited units	1	1, 2	1, 2, 3	1, 2	1, 2, 3, 4	1, 2, 3

The probability of any unit being in service is:

$$P = \frac{\mu}{\mu+\lambda} = \frac{99}{99+1} = 0.99$$

The probability of any unit being on forced outage is:

$$q = \frac{\lambda}{\mu+\lambda} = \frac{1}{99+1} = 0.01$$

The probability of having a state i defined by X units in service and (n-x) units on forced outage out of n units operating in parallel at a time is

$$P_i = 0.99^{x} \times 0.01^{(n-x)}$$

Considering now the first time period (12 m.n. to 4 a.m.). Number of committed units as given in the economical UC table = 1 with this unit operating there is only two possible states, either the unit is available or unavailable.

The probability of the first state $P_1 = 0.99$

The probability of the second state $P_2 = 0.01$

With the unit available, running capacity (12 MW) is greater than the load, therefore $r_1 = 0$. In the second state, the unit is not available, therefore $r_2 = 1$

The security index for this period is:

$$S = 0.99 \times 0 + 0.01 \times 1 = 0.01 > 0.005 \text{ (MTIL)}$$

Thus unit 1 alone supplying the 5 MW load fails to satisfy the prescribed security criterion. In order to obtain optimal and yet secure UC, it is necessary to run the next most economical unit, i.e. unit 2 along with unit 1.

With both unit 1 and 2 operating, there will be four different possible states: - Both units are available, both units are onforced outage, unit 1 available and unit 2 unavailable, unit 1 on forced outage and unit 2 available. The probability that the system state causes breach of system security r equals 1 only in the secon state.

$P_1 = 0.99 \times 0.99 = 0.9801$; $P_2 = 0.01 \times 0.01 = 0.0001$

$P_3 = 0.99 \times 0.01 = 0.0099$; $P_4 = 0.01 \times 0.99 = 0.0099$

The corresponding security index for this cas is:

$$S = 0.9801 \times 0 + 0.0001 \times 1 + 0.0099 \times 0 + 0.0099 \times 0$$
$$= 0.0001 < 0.005 \quad (MTIL)$$

Proceeding similarly and checking security functions for the other periods of the load cycle, we obtain the following economical and secure UC table for the considered station to supply the given load according to the shown load cycle. The obtained table is given below:

Period:	12 m.n. - 4 a.m.	4-8	8-12 n.	12 n. - 4 p.m.	4-8	8-12 m.n.
Load, MW:	5	10	15	10	20	15
Units committed:	1, 2	1, 2	1, 2, 3	1, 2	1, 2, 3, 4	1, 2, 3

3 OPTIMUM GENERATION SCHEDULE

For a given system, if the transmission losses are negligible, the total system load can be optimally devided among various generating stations by applying the equal incremental cost criterion given in equation (8) to the cost curves obtained for individual stations as described in section 2.3.5. It is, however, unrealistic to neglect transmission losses particularly when long distance transmission of power is involved. Transmission losses may very from 5 to 15% of the total load of modern electric utilities serving vast areas of relatively load density. Therefore, it is essential to account for losses while developping the economic load dispatch policy.

In this case the objective is to minimize the overall cost of generation as given by the equation.

$$C = \sum_{i=1}^{m} C_i \quad (P_i) \tag{4}$$

At any time under equality constraint of meeting the load demand with trasmission losses, i.e.

$$\sum_{i=1}^{m} P_i - P_D - P_L = 0 \tag{14}$$

where

m = Total number of generating plants,
P_i = Output power of the ith plant,
P_o = System load demand (Summation of load demand at all buses, and
P_L = Total system transmission losses.

To solve this problem, we write the lagrangian as

$$\mathcal{L} = \sum_{i=1}^{m} C_i(P_i) - \lambda \left[\sum_{i=1}^{m} P_i - P_D - P_L\right] \tag{15}$$

for optimum real power dispatch.

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{dC_i}{dP_i} - \lambda - \text{[illegible]} = 0, \quad i = 1, 2, \ldots, m \tag{16}$$

Rearranging equation (16), we have

$$\frac{dc_i/dP_i}{1-\partial P_L/\partial P_i} = \lambda \quad \text{or} \quad \frac{dc_i}{dP_i} L_i = \lambda, \; i = 1, 2, \ldots, m$$

where $$L_i = \frac{1}{1-\partial P_L/\partial P_i}$$

is called the penalty factor of the ith plant.

The Lagrangian multiplier λ is in LE per MWHR, when the generation cost is in LE per hour and power is in MW.

Equation (17) shows that minimum generation cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for all the plants.

The (m+1) variables ($P_1, P_2, \ldots, P_m$, can be obtained by solving the m optimal dispatch equations (17) together with the power balance equation (14). The partial derivative $\partial P_L/\partial P_i$ is referred to as the incremental transmission loss (ITL), associated with the ith generating station. Equation (17) can also be written in the alternative form.

$$(IC)_i = \lambda \left[1-(ITL)_i\right], \; i = 1, 2, \ldots, m \qquad (18)$$

which is referred to as the exact coordination equation.

To solve the optimum load shceduling problem, it is necessary to compute ITL for all plants. Therfore, before starting the solution, we have to determine the functional dependance of transmission loss on real powers of generating plants, i.e. to determine the function.

$$P_L = f(P_1, P_2, \ldots, P_m) \qquad (19)$$

3.2. GENERAL TRANSMISSION LOSS FORMULA:

The network losses can be obtained by simply adding the fus power at all fuses with a noded voltage Vi at the node i and nodal injected power SI, Current Ii at the same node, the total active and reactive power loss PL - JCL in the network is given by:

$$P_L + jG_L = \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} V_i \overset{*}{I_i} \qquad (20)$$

The last sum can be written as the vector product $V^T_{bus} \; \overset{*}{I}_{bus}$,

Where: (21)

$$V^T_{bus} = [V_1, V_2, \ldots V_n]$$

$$\text{and} \quad I^*_{bus} = \begin{bmatrix} I_1^* \\ I_2^* \\ \vdots \\ I_n^* \end{bmatrix} \quad \text{is the cojugate of the bus current vector } I \text{ bus.} \qquad (22)$$

But $V_{bus} = I_{bus} \; Z_{bus}$, and $V^T_{bus} = I^T_{bus} + Z^T_{bus}$, where

Z_{bus} is the bus impedance matrix of the network

$$Z_{bus} = R_{bus} + jX_{bus} = \begin{bmatrix} r_{11} & r_{12} \cdots r_{1n} \\ r_{21} & r_{22} \cdots r_{2n} \\ \vdots & \vdots \\ r_{n1} & r_{n2} \cdots r_{nn} \end{bmatrix} + j \begin{bmatrix} x_{11} & x_{12} \cdots x_{1n} \\ x_{21} & x_{22} \cdots x_{2n} \\ \vdots & \vdots \\ x_{n1} & x_{n2} \cdots x_{nn} \end{bmatrix} \qquad (21)$$

Similarly, the bus current vector can be written as a sum of a real and reactive component vector as follows:

$$I_{bus} = I_p + jI_q = \begin{bmatrix} I_{p1} \\ I_{p2} \\ \vdots \\ I_{pn} \end{bmatrix} + j \begin{bmatrix} I_{q1} \\ I_{q2} \\ \vdots \\ I_{qn} \end{bmatrix} \qquad (24)$$

Substituting all these equations in equation (20) we get:

$$P_L + jG_L = I^T_{bus} \; Z^T_{bus} \; \overset{*}{I}_{bus} = I^T_{bus} \; Z_{bus} \; \overset{*}{I}_{bus} \qquad (25)$$

$$= (I_p + jI_q)^T (R + jX)(I_p - jI_q)$$

Z^T_{bus} is taken equal to Z bus because it is a symmetrical matrie.

Taking the real part of both sides of equation (25) ,we get:

$$PL = I_p^T R I_p + I_p^T X I_q + I_q^T R I_q - I_q^T X I_p \qquad (26)$$

It is easy to proove that the second and fourth terms are equal because the matrie X is symmetrical, therefore Equ.(26) reduces to

$$PL = I_p^T R I_p + I_q^T R I_q$$

Returning back to the index notation PL can be written

$$PL = \sum_{\substack{j=1 \\ k=1}}^{n} r_{jk} \left(I_{pj} \; I_{pk} + I_{qj} \; I_{qk} \right)$$

To express the total power loss in terms of bus powers and bus voltage instead of expressing it in terms of bus currents as given in Equ.(28),we will proceed as fol.:

$$I_i = I_{pi} + j I_{qi} = \frac{S_i^*}{V_i^*} = \frac{P_i - j Q_i}{|V_i| \left(\cos \delta_i - j \sin \delta_i \right)}$$

$$= \frac{P_i - j Q_i}{|V_i|} \left(\cos \delta_i + j \sin \delta_i \right)$$

$$\text{or } I_{pi} + j I_{qi} = \frac{P_i \cos \delta_i + Q_i}{|V_i|} + j \, \frac{P_i \sin \delta_i - Q \sin \delta_i}{|V_i|} \qquad (29)$$

Where δ_i is the angle of V_i with respect to the reference bus voltage(i.e. slack bus voltage). By seperating the imaginary and real parts of Eq. (19), we get:

$$I_{pi} = \frac{1}{|V_i|} \left(P_i \cos \delta_i + Q_i \sin \delta_i \right)$$

$$I_{qi} = \frac{1}{|V_i|} \left(P_i \sin \delta_i - Q \cos \delta_i \right) \qquad (30)$$

Substituting these expressions for the currents into the Eq,(28), we get PL,after some algebraic operations, as:

$$PL = \sum_{\substack{j=1 \\ k=1}}^{n} \left[\alpha_{jk} \left(P_j P_k + Q_i Q_k \right) + \beta_{jk} \left(Q_j P_k - P_j Q_k \right) \right] \qquad (31)$$

Where:

$$\alpha_{jk} = \frac{r_{jk}}{|V_j||V_k|} \cos \left(\delta_j - \delta_k \right)$$

$$\beta_{jk} = \frac{r_{jk}}{|V_j||V_k|} \sin \left(\delta_j - \delta_k \right)$$

3.3. INCREMENTAL TRANSMISSION LOSS ITL:

The incremental transmission loss (ITL) i for the power plant i delivering an output power P_{ai} is given by:

$$(ITL)_i = \frac{\partial P_L}{\partial P_{ai}} = \sum_{\substack{j=1 \\ k=1}}^{m} \frac{\partial}{\partial P_i} \left(\alpha_{jk} (P_j P_k + Q_j Q_k) + \beta_{jk} (Q_j P_k - P_j Q_k) \right] \quad (33)$$

∂P_{ai} is replaced by ∂P_i because the power balance equation at node i, is

$$P_{ai} = P_i + P_{Di}$$

Where PDi is the demand power at this bus and is kept constant so that

$$\partial P_{Di} = 0 \quad \text{and} \quad \partial P_{ai} = \partial P_i$$

Manipulating Eq.(33) and rearranging it (ITL) i can be changed to the form:

$$(ITL)_i = 2 \sum_{k=1}^{m} (P_k \alpha_{ik} - Q_k \beta_{ik}) - \sum_{\substack{j=1 \\ k=1}}^{m} \left[(P_j P_k + Q_j Q_k) \frac{\partial \alpha_{jk}}{\partial P_i} - (P_j Q_L - Q_j P_k) \frac{\partial \beta_{jk}}{\partial P_i} \right] \quad (34)$$

Also it can be shown that

$$\frac{\partial \alpha_{jk}}{\partial P_i} = \frac{r_{jk} \sin (\delta_j - \delta_k)}{|V_i||V_j||V_k|} \left[\frac{1}{|Y_{ij}||V_j| \sin (\delta_j - \delta_i + \theta_{ij})} - \frac{1}{|Y_{ik}||V_k| \sin (\delta_k - \delta_i + \theta_{ik})} \right] \quad (35)$$

and

$$\frac{\partial \beta_{jk}}{\partial P_i} = \frac{r_{jk} \cos (\delta_j - \delta_k)}{|V_i||V_j||V_k|} \left[\frac{1}{|Y_{ik}||V_k| \sin (\delta_k - \delta_i + \theta_{ik})} - \frac{1}{|Y_{ij}||V_j| \sin (\delta_j - \delta_i - \theta_{ij})} \right]$$

Where: $Y_{ik} = |Y_{ik}| \underline{/\theta_{ik}}$ and $Y_{ij} = |Y_{ij}| \underline{/\theta_{ij}}$

are the elements of the bus admittance matrix of the network having the indices iK and ij respectively.

Equations (35) together with equation (34) permit us to compute the ITLs from the knowledge of bus voltages and bus powers.

Experience with typical system parameters has show that the contribution of the doublesum term in Eq.(34) to ITLs, is usually insignificant, which may allow us to neglect this term to get the following approximate but time saving formula for (ITL).

$$(ITL)_i \approx 2 \sum_{k=1}^{m} (P_k \alpha_{ik} - Q_k \beta_{ik}) \quad (36)$$

3.4. Digital Computer Application to Solve the Optimum Generation Scheduling Problem:

The problem here is somewhat complicated because the "Optimum dispatch Equations" and the "Power balance equation" contain loss terms which themselves are functions of all the individual generator outputs. This makes the dispatch equations coupled , i.e. each equation is a function of all generator output powers. Figure 7 shows a flow chart for a proposed digital computer program to solve the optimum dispatching problem for a lossy system. The basic computation steps are as follows:

1. Giving available information about all load demands SDi, specified voltage magnitudes/Vi/ for control buses and assumed initial values of generator powers, a solution to the load flow problem is then performed.
2. Having found, from the output of load flow solution,information about all bus voltages,power angles,generator powers,the (ITL)i can be computed which permit us to start an iterative process by to solve the optimal dispatch equations to find a first set of optimum generator outputs.
3. An initial guess is made regarding the generator output ,we have no assurance that htese initial values are optimal in any sense.
4. The generator outputs are then readjusted in accordance with computed optimum valves, then we solve the load flow problem.
5. The inner and outer iterative processes will continue until a solution within the specified accuracy is reached.

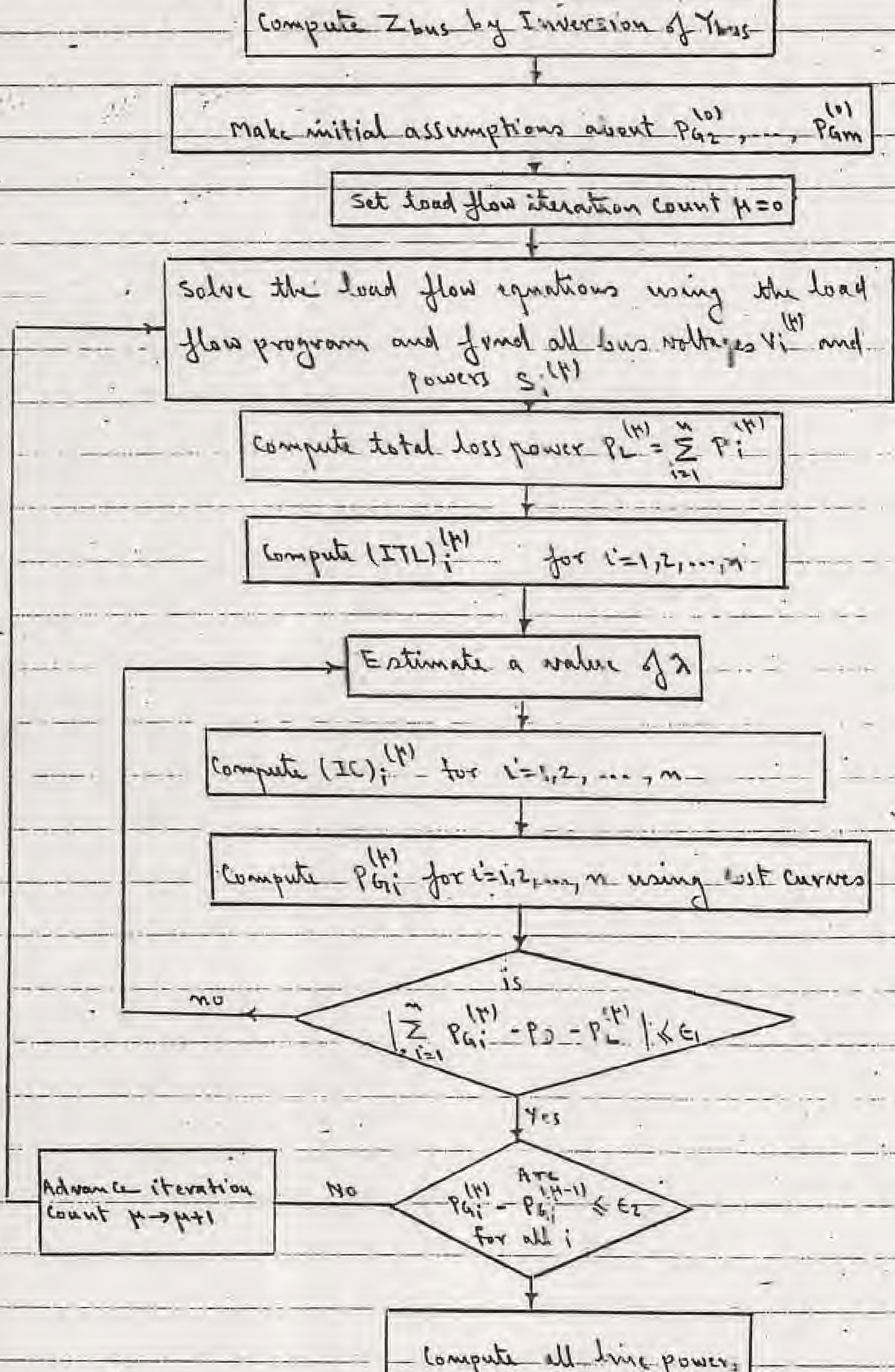


Fig. 7 Flow diagram for computation of the optimum dispatch problem with consideration of losses